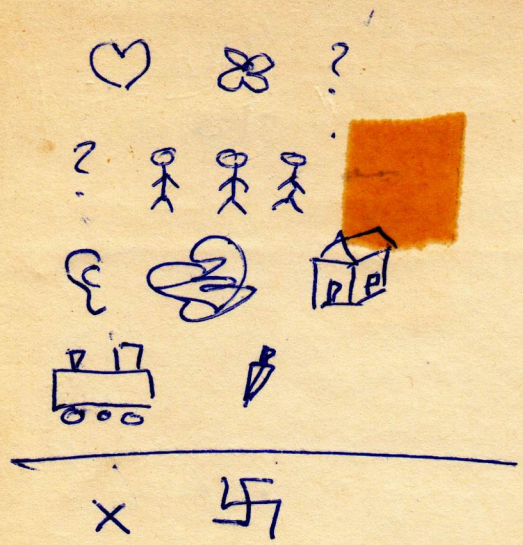


Dragi supruže, kako si?
Kako naše troje djece?
Čuo sam da šaraš oko kuće.
Kada dođem, ubit ću te.

Mujo iz Trp. Njemačke

→ šuvarenje – priđanje gluposti

→ Spiljak i Dolac – dva sa 2

→ supromodljivost
→ poluvodič
→ lasar
→ fotomultiplikator

Br. 1. D32,154 (1985

ISV

I Motivacija za Q.M.

1) zračenje crnog tijela (apsorbcija = 1, transmisija = 0, refleksija = 0)

UAH-I str. 35

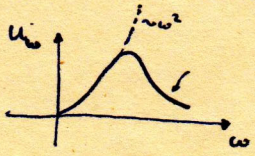
Klasična statistič. fizika $\frac{U}{V} = u = \int_0^\infty \left(\frac{du}{d\omega}\right) d\omega$ $\quad u_\omega = \frac{du}{d\omega} = \frac{kT}{\pi^2 c^2}\omega^2$ Jeans - Rayleigh

→ ultravioletna katastrofa

Kvantna statistička fizika M. Planck energ. nivoi harm. oscilatora

$E_n = n\hbar\omega \quad n = 0, 1, 2, \ldots$

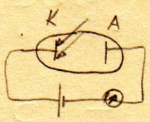
$$u_\omega = \frac{\hbar\omega}{e^{\frac{\hbar\omega}{kT}} - 1}$$



$\Rightarrow u \sim T^4$

$\hbar$ -

2) Fotoelektrički efekt



energija izlažnja elektrona ne ovisi o intenzitetu svjetlosti, već o boji (ν)

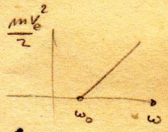
Einstein 1905 - hipoteza o kvantima svjetlosti (fotoni) energija $\boxed{\hbar\omega = E}$ ($h\nu$)

$$\hbar\omega + eV_i = \frac{mv_e^2}{2} + eV_a + E_s$$

eV_i: raspoloživa kin. energija u metalu

eV_a: energija ponovnog ubrzavanja

E_s: gubitak energije elast. sudaranja do ponovnog



$\Rightarrow \exists$ minimalna energija - prag fotoefekta $J = \hbar\omega_0$

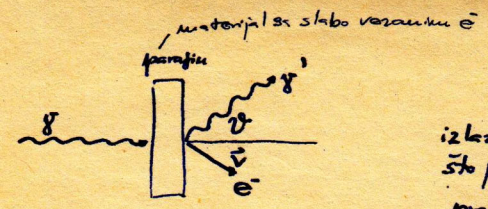
- foton ima impuls $p = m'c = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$ $\quad \boxed{\vec{p} = \hbar\vec{k}}$ $\quad |\vec{k}| = \frac{2\pi}{\lambda}$

$E = mc^2$

Korpuskula E, p ($E = cp$)

val ν, λ

3) Comptonov efekt



izlazni foton je to manje energije, što je jače otklonjen

promjena $\nu > \nu'$ - klasično nerazjašnjivo

pretp. e^- miruje

$$\vec{p} = \vec{p}' + \frac{m_0 \vec{v}}{\sqrt{1-\frac{v^2}{c^2}}} \qquad \hbar\omega + m_0c^2 = \hbar\omega' + \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

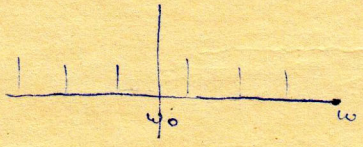
$$p = \frac{h}{\lambda} = \frac{\hbar\omega}{c} \qquad p' = \frac{\hbar\omega'}{c} \qquad \Rightarrow \quad \Delta\lambda = \lambda' - \lambda = 2\lambda_c \sin^2\frac{\vartheta}{2}$$

Comptonova valna duljina $\lambda_c = \frac{h}{m_0 c} = 2{,}42 \cdot 10^{-8}$ cm

4) Ramanov efekt

- monokromatski el.mg. val pada na tekućine ili plinove

$\Rightarrow$ izlazni el.mg. val se pojavljuje i sa drugim frekvencijama koje su više ili niže od početne frekvencije



$\rightarrow$ kvantizirane energije molekula dovodjamo kvantna stanja molekule

5) Spektralna analiza el. mg. zračenja atoma

exp:

Balmer	$\nu = Z^2 R \left(\frac{1}{2^2} - \frac{1}{n^2}\right)$	$n = 3, 4, \ldots$	$R = 109\,677{,}58\ \mathrm{cm}^{-1}$
Lyman	$\nu = Z^2 R \left(\frac{1}{1^2} - \frac{1}{n^2}\right)$	$n = 2, 3, \ldots$	
Paschen	$\nu = Z^2 R \left(\frac{1}{3^2} - \frac{1}{u^2}\right)$	$u = 4, 5, \ldots$	
Bracket	$\nu = Z^2 R \left(\frac{1}{4^2} - \frac{1}{n^2}\right)$	$n = 5, 6, \ldots$	
Pfund	$\nu = Z^2 R \left(\frac{1}{5^2} - \frac{1}{n^2}\right)$	$n = 6, 7, \ldots$	

$\rightarrow$ pokušaj mikroskopskog objašnjenja - uloga atomskog modela

Thompson 1903 - elektron pliva u jednolikoj nabijenoj poz. kugli $r = 10^{-8}$ cm

Rutherford 1911 - raspršenje α čestica $1f = 10^{-13}$ cm

Maxwell $\Rightarrow$ titranje el. dipola $\rightarrow$ zračenje

$$E = -\frac{Ze^2}{2r} \qquad |E| \downarrow \Rightarrow r \downarrow$$

neharmonijsko gibanje $x(t) = \sum_{k=0}^{\infty} a_k e^{ik\omega t}$

$\omega, 2\omega, \ldots k\omega$ nekoristi se

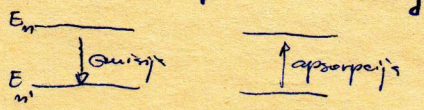
$\rightarrow$ poremećaje u spektru zbog sudaranja se ne spominju

N. Bohr-ovi postulati (1913)

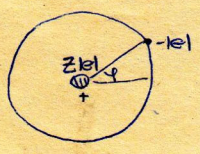
1) boravak e^- u stacionarnim energetskim stanjima $E_0, E_1, E_2, \ldots$

2) e^- u jednom stacionarnom energetskom stanju ne emitira el. mg. val
do emisije dolazi tek kad elektron prelazi iz višeg energ. stanja u niže

$$\omega_{nn'} = \frac{E_n - E_{n'}}{\hbar}$$



Somorfold-Wilsonovi uvjeti kvantizacije $\oint p_k \, dq_k = n_k \cdot h$



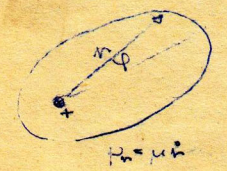
$$p_\varphi = \overset{|\vec{r}\times\vec{p}|}{mrv} = n\hbar$$

$$V_n = \frac{Ze^2}{n\hbar} \qquad r_n = \frac{n^2\hbar^2}{mZe^2} \qquad E_n = -\frac{mZ^2e^4}{2n^2\hbar^2} \qquad \omega_{nn'} = Z^2\frac{me^4}{2\hbar^3}\left(\frac{1}{n'^2} - \frac{1}{n^2}\right)$$

$\omega = 2\pi\nu$

→ problem 2 tijela reducirana masa $\frac{1}{\mu} = \frac{1}{m} + \frac{1}{M}$ $E_n = -\frac{\left(\frac{\mu}{m}\right) mZ^2e^4}{2n^2\hbar^2}$ $R_0 = 109\,737{,}3\ \mathrm{cm}^{-1}$

$$R_0' = \frac{1}{\left(1 + \frac{m}{M}\right)} R_0$$



$$E = -\frac{\mu Z^2 e^4}{2\hbar^2 (n_r + n_\varphi)^2}$$

$n = n_r + n_\varphi$ — radijalni kv. broj; azimutalni kvantni broj

$b = a\frac{n_\varphi}{n}$ $n_r = 0$ Kružnica

$p_r = \mu \dot{r}$ $\oint p_r \, dr = n_r h$ $n_r = 0, 1, 2, \ldots$ $p_\varphi = n_\varphi \cdot \hbar$ $n_\varphi = 1, 2, 3$

$n = 1 \Rightarrow n_r = 0 \quad n_\varphi = 1$

$n = 2 \Rightarrow n_r = 0 \quad n_\varphi = 1$
$n_r = 1 \quad n_\varphi = 1$

$n = 3 \Rightarrow n_r = 0 \quad n_\varphi = 3$
$n_r = 1 \quad n_\varphi = 2$
$n_r = 2 \quad n_\varphi = 1$

→ Bohr-ova korekcija s kružnice na elipsu ne daje dovoljan broj stanja s istom energijom u odnosu na linije (n stanja iste energije E_n)

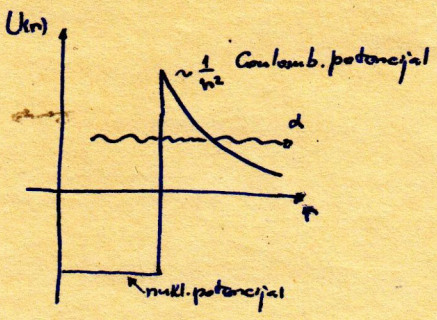
exp. degeneracija E_n $2n^2$

→ Bohr-ova teorija na He ne daje dobre rezultate

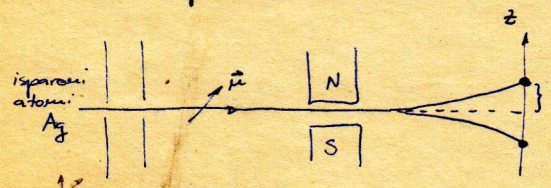
→ u Bohr-ovoj teoriji radimo s pojmom staze → u ∀ trenutku bi znali p, r

→ relacije neodređenosti

6) α-raspad



7) Stern-Gerlach (1922)



$U_{pot} = -\vec{\mu}\cdot\vec{H}$

$\vec{F} = -\vec{\nabla} U = |\vec{\mu}|\cos\vartheta\, \vec{\nabla}|\vec{H}|$ $\qquad \frac{dH}{dz} \neq 0$

$= \vec{k}\, |\vec{\mu}| \frac{dH}{dz} \cos\vartheta$

$\Rightarrow$ sila kontinuirano $\quad -|\vec{\mu}|\frac{dH}{dz}, \dots |\vec{\mu}|\frac{dH}{dz}$

$l = 0, 1, 2, \dots$

$m = -l, -l+1, \dots +l$

$l = 1/2 \quad m = -1/2, 1/2$

$l = 3/2$

$\Rightarrow$ polucijeli spin $\quad S_z = \Gamma_z = \pm\frac{\hbar}{2}$

8)

$N_{12} = N_1 + N_2$

otsutnost interferencije

I_{12} – intenzitet vala tj. energija titranja

$h_{12}^2 = I_{12} \neq I_1 + I_2 = h_1^2 + h_2^2$

$h_{12} = h_1 + h_2$ ← vremenski

$a_{12}^2 = N_{12} \neq N_1 + N_2 = a_1^2 + a_2^2$

$a_{12} = a_1 + a_2$

Pušćana zrna	Valovi uopće	Elektroni, fotoni
Dolaze jedno po jedno	Mogu biti bilo koje veličine	Dolaze fotoni po jedan
Mjera vjerojatnosti dolaska $N_{12} = N_1 + N_2$	Mjera intenziteta valova $I_{12} \neq I_1 + I_2$	Mjera vjerojatnosti dolaska $N_{12} \neq N_1 + N_2$
Nema interferencije	Interferencija postoji	Interferencija postoji

→ mijenjamo 2 : 1 & 2 ništa ne upada
1 zatvorim upada

→ ∀ put prije otkrivanja detektora bijeme 1 V 2
Sačuvajmo priroda u kut !!

detektor 2' $N_{12} = N_1 + N_2$ $\Rightarrow$ priroda iznenađena

$\Rightarrow$ svjetlost utječe na rezultat

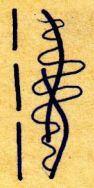
→ svjetlost ostaje N_1 N_2 el. kje nisam vidio

i-nared $N_1 + N_2$ $\qquad N_{12}$ raspored

→ skrivene varijable – kontradikcija sa interferencijom
„Čak ni sama priroda ne zna kuda će proći elektron"

Interf. & ogib Kathodnih zraka

II



$\Rightarrow$ teorija valova materije

de Broglie-evi valovi (1924)

čestica	val
$E = h\nu = \hbar\omega$	ν
$\vec{p} = \frac{h}{\lambda}\vec{k}_0 = \hbar \cdot \vec{k}$	λ

$\omega = 2\pi\nu$

$|\vec{k}| = \frac{2\pi}{\lambda}$

$$E = \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}} = \sqrt{p^2c^2 + m_0^2c^4}$$

$$\left(\frac{E}{c}\right)^2 - p^2 = m_0^2c^2$$

$$\left(\frac{\omega}{c}\right)^2 - k^2 = \left(\frac{m_0 c}{\hbar}\right)^2$$

1-dim. ravni val

$$\psi(x,t) = A\cos(kx - \omega t) = A\cos 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)$$

ili $\psi(x,t) = A e^{i(kx - \omega t)}$

3. dim. ravni val

$$\psi(\vec{r},t) = A e^{i(\vec{k}\vec{r} - \omega t)} = A e^{i\left(\frac{\vec{p}\vec{r}}{\hbar} - \frac{E}{\hbar}t\right)}$$

val materije (?)

$$\psi(\vec{r},t) = \int F(\vec{p})\, e^{\frac{i}{\hbar}(\vec{p}\vec{r} - Et)}\, d^3p$$

(slobodna čestica) nerelativistička $E \simeq \frac{mv^2}{2} = \frac{p^2}{2m}$

$$i\hbar\frac{\partial}{\partial t}\psi(\vec{r},t) = \int \hbar E\, e^{\frac{i}{\hbar}(\vec{p}\vec{r} - Et)}\, d^3p$$

$$-\hbar^2\Delta_r\psi(\vec{r},t) = \int \vec{p}^2 F(\vec{p})\, e^{\frac{i}{\hbar}(\vec{p}\vec{r} - Et)}\, d^3p$$

$$\Rightarrow \quad i\hbar\frac{\partial}{\partial t}\psi(\vec{r},t) = -\frac{\hbar^2}{2m}\Delta_r\psi(\vec{r},t)$$

princip korespondencije

slobodna čestica (energ. izolirana)
nerelativistička

$$H = E \qquad H = \frac{\vec{p}^2}{2m}$$

$$H\psi = E\psi$$

$$E \to i\hbar \frac{\partial}{\partial t}$$

$$\vec{p} \to \frac{\hbar}{i} \vec{\nabla}$$

$$\frac{1}{2m}\left(\frac{\hbar}{i}\right)^2 \vec{\nabla}^2 \psi(\vec{r},t) = i\hbar \frac{\partial \psi}{\partial t} \qquad \hat{H} = -\frac{\hbar^2}{2m}\Delta_r$$

$$\hat{H}\psi(\vec{r},t) = i\hbar \frac{\partial \psi}{\partial t}$$ Schröd. jba.

slobodna čestica (energ. izolirana)
relativistička

$$\left(\frac{E}{c}\right)^2 - p^2 = m_0^2 c^2$$

$$\left[\frac{1}{c^2}(i\hbar)^2 \frac{\partial^2}{\partial t^2} - \left(\frac{\hbar}{i}\right)^2 \vec{\nabla}^2\right]\psi = m_0^2 c^2 \psi$$

→ relativistička Q.M.

$$\left(\Delta_r - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\psi(\vec{r},t) - \left(\frac{m_0 c}{\hbar}\right)^2 \psi(\vec{r},t) = 0$$ Klein-Gordon jednadžba

fotoni:
$m_0 = 0$ $\Rightarrow$ $\left(\Delta_r - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\psi(\vec{r},t) = 0$ valna jba. za svjetlost

Vremenski neovisna Schröd. jba.

$$H = \frac{\vec{p}^2}{2m} + U(\vec{r}) = E \qquad \frac{\partial H}{\partial t} = 0 \qquad \hat{H} = -\frac{\hbar^2}{2m}\Delta_r + U(\vec{r})$$

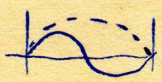
$$\Psi(\vec{r},t) = \psi(\vec{r})\varphi(t) \quad \& \quad i\hbar\frac{\partial \Psi}{\partial t} = \hat{H}\Psi \quad \Rightarrow \quad i\hbar\frac{\partial \varphi}{\partial t}\cdot\psi = \varphi\hat{H}\psi$$

$$\frac{\hat{H}\psi}{\psi} = i\hbar\frac{\dot{\varphi}}{\varphi} = E = \text{const.}$$

$$\Rightarrow \varphi(t) = e^{-i\frac{E}{\hbar}t}$$

$$\Rightarrow \Psi(\vec{r},t) = \psi(\vec{r})\, e^{\frac{i}{\hbar}Et}$$

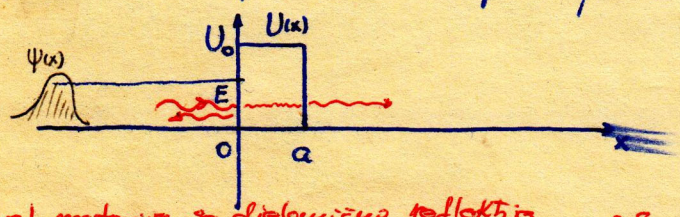
stacionarni val materije



$$\hat{H}\psi(\vec{r}) = E\psi(\vec{r})$$

zahtjevi regularnosti na $\psi(\vec{r})$: 1° kontinuiranost
2° konačnost
3° jednoznačnost

Primjer: čestica koja nailazi na potencijalnu barijeru (1-dim)



→ val materije se djelomično reflektira i djelomično transmitira !!

$$\hat{H}\psi(x) = E\psi(x)$$

$$\left[-\frac{\hbar^2}{2m}\Delta_r + U(x)\right]\psi(x) = E\psi(x)$$

$$\psi''(x) + \frac{2m}{\hbar^2}(E - U(x))\psi(x) = 0 \qquad U(x) = \begin{cases} 0 & x<0 \\ U_0 & 0<x<a \\ 0 & x>a \end{cases}$$

$x \leq 0$ $\quad U(x) = 0 \quad \psi''(x) + k^2\psi(x) = 0 \quad k = \frac{1}{\hbar}\sqrt{2mE}$

$$\psi(x) = e^{ikx} + Ae^{-ikx}$$

$x \in (0,a)$ $\quad U(x) = U_0 \quad \psi''(x) - \frac{2m}{\hbar^2}(U_0 - E)\psi(x) = 0 \quad k_0 = \frac{1}{\hbar}\sqrt{2m(U_0 - E)}$

$$\psi(x) = Be^{k_0 x} + Ce^{-k_0 x}$$

$x > a$ $\quad U(x) = 0 \quad \psi''(x) + k^2\psi(x) = 0$

$$\psi(x) = De^{ikx}$$

odredivanje A, B, C, D

$x = 0$ $\quad 1 + A = B + C$

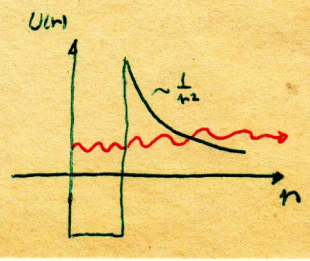
$k(1 - A) = -ik_0(B - C)$

$x = a$ $\quad Be^{k_0 a} + Ce^{-k_0 a} = De^{ika}$

$k_0(Be^{k_0 a} - Ce^{-k_0 a}) = ikDe^{ika}$

tunel efekt

α-raspad



→ Fizikalno značenje valne fje. $\Psi(\vec{r},t)$ tj. $\psi(\vec{r})$

→ stacionarni slučaj $|\Psi(\vec{r},t)|^2 = |\psi(\vec{r})|^2$ ⇒ smisao treba tražiti u $|\Psi(\vec{r},t)|^2 = \Psi\Psi^*$

pretp. Schrödinger : gustoća vala materije (naboja)

$$\rho = e|\Psi|^2 \qquad \Rightarrow \quad e = \int \rho\, d^3r = e\int |\Psi|^2 d^3r$$

$$\int |\Psi|^2 dV = 1$$

neodrživost 1° $|\Psi|^2$ velika prostorna gustoćnost ⇒ $\bar{e}$ bi se raoplinio

2° val materije bacimo na potencijalni bedem ⇒ djelomično se reflektira i transmitira apsurd!

→ Statistička interpretacija valne fje. valne mehanike (M. Born)

$w = |\Psi(\vec{r},t)|^2$ (prostorna) gustoća vjerojatnosti da $\bar{e}$ nađemo negdje u prostoru

⇒ valovi materije gube očigledno značenje, koje su im pripisali de Broglie & Schrödinger

⇒ „valovi vjerojatnosti"

→ interferencija i ogib katodnih zraka : tamna i svjetla mjesta ne znače da su se tamo $\bar{e}$ poništili ili pojačali, već kako su često na da mjesta padali $\bar{e}$ katodnih zraka

Sačuvanje norme (ukupne vjerojatnosti) u vremenu

$$N(t) = \int |\psi|^2 d^3r \qquad \frac{dN(t)}{dt} = 0 \quad \Rightarrow \quad \frac{\partial}{\partial t} |\psi(\vec{r},t)|^2 = 0$$

$$\left.\begin{array}{l} i\hbar \frac{\partial}{\partial t}\psi = H\psi \\ i\hbar \frac{\partial}{\partial t}\psi^* = -(H\psi)^* \end{array}\right\} \Rightarrow \frac{\partial}{\partial t}|\psi|^2 = \frac{1}{i\hbar}\left[\psi^*(H\psi) - (H\psi)^*\psi\right] \Bigg\} \Rightarrow H\text{- hermi. op.}$$

$$\int \psi\psi^* d^3r = \langle\psi|\psi\rangle$$

F - fizikalna veličina $\longrightarrow$ $\hat{F}$ - operator $\qquad \frac{d}{dt}\hat{F}(t) = \frac{1}{i\hbar}[\hat{F},\hat{H}] + \frac{\partial \hat{F}}{\partial t}$

$$\langle F\rangle = \int \psi^* \hat{F}\psi d^3r = \langle\psi|\hat{F}|\psi\rangle$$

srednja vrijednost koja ima fizikalnog smisla - ono što se može mjeriti

$$\hat{F}\psi = f\psi \quad \Rightarrow \quad \langle F\rangle = f = \langle\psi|\hat{F}|\psi\rangle \in \mathbb{R} \quad \Rightarrow \quad F\text{ - hermi. op.}$$

str 295

$$A = \begin{bmatrix} 1 & 8 & -7 \\ 2 & 7 & -5 \\ -4 & 4 & -8 \end{bmatrix} \qquad x_1 = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} = 1\cdot e_1 - 1\cdot e_2 - 1\cdot e_3 \qquad x_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \qquad x_3 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$

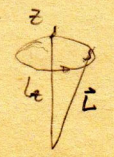
$$Ax_1 = 0\cdot x_1 \qquad Ax_2 = 9\cdot x_2 \qquad Ax_3 = -9x_3$$

$$[\hat{F},\hat{G}] = i\hbar \qquad \Delta F = \sqrt{\langle\hat{F}^2\rangle - \langle\hat{F}\rangle^2} \quad \text{dovjerja}$$

$$[x,p_x] = i\hbar I \qquad \frac{\hbar}{i}\left(x\frac{\partial\psi}{\partial x} - \frac{\partial}{\partial x}(x\psi)\right) = -\frac{\hbar}{i}(-\psi) \qquad -\psi - x\frac{\partial\psi}{\partial x}$$

$$\Rightarrow \Delta F\cdot\Delta G \geq \frac{\hbar}{2} \quad \text{relacije neodređenosti} \qquad \Rightarrow \Delta x\cdot\Delta p_x \geq \frac{\hbar}{2}$$

$$\vec{L} = \vec{r}\times\vec{p} \qquad [L_x,L_y] = i\hbar L_z \qquad [L_y,L_z] = i\hbar L_x \qquad [L_z,L_x] = i\hbar L_y$$



precesija

$$\vec{L}^2\psi_{nlm} = -\hbar l(l+1)\psi_{nlm}$$

$$L_z\psi_{nlm} = m\hbar\psi_{nlm}$$

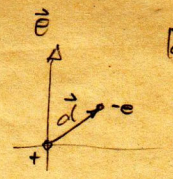
Primjer: H_0 ima dva dvostruko degenerirana nivoa

$$H_0 \mu_n^{(\alpha)} = E_n^{(0)} \mu_n^{(\alpha)} \qquad n=1,2 \quad \alpha=1,2$$

razmak između nivoa je $E_2^{(0)} - E_1^{(0)} = \varepsilon$.

Odrediti pomak nivoa koji nastaje uključivši smetnju V koja ima matrične elemente

$$\langle n\alpha|V|n'\alpha'\rangle \equiv V^{n\alpha}_{n'\alpha'} \equiv \int \mu_n^{(\alpha)*} V \mu_{n'}^{(\alpha')} d\tau = \begin{cases} \frac{\varepsilon}{8} & n \neq n' \\ 0 & n = n' \end{cases}$$



$|\vec{d}| = |e|\,l$

$E_p = -\vec{d}\,\vec{E}$

npr. Stark-ov efekt

Tatjana: 521-150

$$H_0 \mu_n^{(\alpha)} = E_n^{(0)} \mu_n^{(\alpha)}$$

$$H = H_0 + V$$

$$H\psi = W\psi \qquad \psi = \sum_{n=1}^{2} \sum_{\alpha=1}^{2} c_{n\alpha} \mu_n^{(\alpha)}$$

$$\sum_{n,\alpha} c_{n\alpha} \Big\{ \underbrace{H_0 \mu_n^{(\alpha)}}_{= E_n^{(0)} \mu_n^{(\alpha)}} + V \mu_n^{(\alpha)} - W \mu_n^{(\alpha)} \Big\} = 0$$

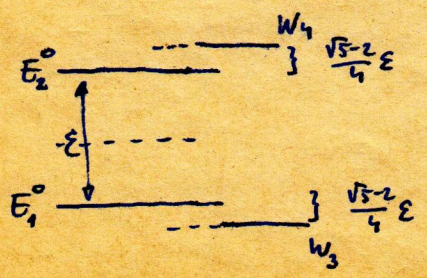
$$\sum_{n,\alpha} c_{n\alpha} \Big\{ (E_n^{(0)} - W)\delta_{nn'}\delta_{\alpha\alpha'} + \langle n'\alpha'|V|n\alpha\rangle \Big\} = 0 \qquad \begin{matrix} n'=1,2 \\ \alpha'=1,2 \end{matrix}$$

$$\begin{vmatrix} E_1^{(0)} - W & 0 & \frac{\varepsilon}{8} & \frac{\varepsilon}{8} \\ 0 & E_1^{(0)} - W & \frac{\varepsilon}{8} & \frac{\varepsilon}{8} \\ \frac{\varepsilon}{8} & \frac{\varepsilon}{8} & E_2^{(0)} - W & 0 \\ \frac{\varepsilon}{8} & \frac{\varepsilon}{8} & 0 & E_2^{(0)} - W \end{vmatrix} = 0 \quad \Rightarrow \quad (E_1^{(0)} - W)(E_2^{(0)} - W)\Big[(E_1^{(0)} - W)(E_2^{(0)} - W) - \frac{\varepsilon^2}{16}\Big] = 0$$

$$W_1 = E_1^{(0)} \quad \& \quad W_2 = E_2^{(0)}$$

$$W^2 - W(E_2^{(0)} + E_1^{(0)}) + E_1^{(0)} E_2^{(0)} - \frac{\varepsilon^2}{16} = 0$$

$$\Rightarrow W_{3,4} = \frac{E_1^{(0)} + E_2^{(0)}}{2} \pm \frac{\sqrt{5}}{4}\varepsilon$$



UAM II 10.12 – 13.42 6.4.73.

Zadaci:

(20)

① Naći koef. transm. čestice kroz pot. barijeru oblika $U(x)$ – približ. metodu $U(x)=\begin{cases}\alpha x+U_0\\ -\alpha x+U_0\end{cases}$

(18)

② Iz Hers. rel. neodređenosti ocijeniti energiju osn. stanja čestice ako se ona nalazi u potenc. polju $U(x)$

$$U(x)=\alpha^2x^2-\frac{\beta}{x^2} \qquad \Delta x\cdot\Delta p\geq\frac{\hbar}{2}$$

(24)

③ Odred. varijac. metodom energ. osn. stanja čestice za potenc. polje $U(r)=\frac{A}{r^2}-\frac{B}{r}$

probna fja. $u_\alpha(r)=e^{-\alpha r}$

(20)

④ Odrediti energiju fermionskog plina, ako je gustoća energije data sa izrazom

$$g(E)=cE^{3/2}$$

(18)

⑤ Zadana je matrica oper. A u $t=0$ odred. $A(t)$ ako je H sistem dan sa matricom

$$A(t=0)=\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} \qquad H=\begin{pmatrix}\varepsilon & 0\\ 0 & 2\varepsilon\end{pmatrix} \qquad A(t)=?$$

usmeni

ponedj. 8 sati Rudar – teor.

seminar

16.IV 8h

20.IV 8h. IRB

$\frac{d}{dr}(ry)=y+ry'$

$\frac{1}{r}\frac{d^2}{dr^2}(ry)$

$\frac{1}{r}\cdot(y'+y'+ry'')$

$y''+\frac{2}{r}y'$

$\frac{d}{dr}\left(\frac{1}{r}\frac{d}{dr}\right)-\frac{2}{r^2}\frac{d}{dr}+\frac{1}{r}$

$\frac{d^2}{dr^2}+\frac{1}{r}$

9.20 – 12.20

I dio

① Za van der Waals-ov plin čija je jedn. stanja (20)

$$\left(P+\frac{an^2}{V^2}\right)(V-nb)=nRT$$ pomoću t-din.

relacija odrediti ovisnost unutr. energ. U o volumenu V.

ako se smatra da je poznato $U=U(T)-\frac{an^2}{V}$ $V=const.$ (18)

② Za dvodim. idealni plin (u ravnini) odrediti u Maxw. raspodjeli $\bar{V}$, $\overline{V^2}$, V_w $\sqrt{\frac{\pi kT}{2m}}$, $\frac{2kT}{m}$, $\sqrt{\frac{kT}{m}}$

9.50.

③ Odrediti u Maxw. raspodjeli % čestica sa brzinom većom od $\bar{V}$. (18)

10.40 46,57%

④ Izrač. slob. energ. sistema F u kojem se čestice mogu nalaziti na tri nedegenerirana nivoa.

$g_0=g_1=g_2=1$ pri čemu su energije nivoa

$E_0=0$ $E_1=1eV$ $E_2=2eV$, a populacije prva dva nivoa su (20)

$N_0=2\cdot10^{22}$ $N_1=4\cdot10^{22}$

pri čemu je $1eV=1{,}6\cdot10^{-12}$ erg $F=\frac{\ln 7}{\ln 2}\,eV$

⑤ Izračunati $\overline{M_z}$, $\overline{M_z^2}$, $\overline{M^2}$ za anizotropni harm. oscilator čiji je Hamitl. dan sa $H=\frac{p^2}{2m}+\frac{m}{2}(\omega_1^2x^2+\omega_2^2y^2+\omega_3^2z^2)$

11.15.

$\vec{M}=\vec{r}\times\vec{p}$

$$\overline{M^2}=2k^2T^2\left(\frac{1}{\omega_1^2}+\frac{1}{\omega_2^2}+\frac{1}{\omega_3^2}\right)$$

Repit 16.2.73. M.D.

$p_B = \frac{h}{2\pi}$

$-\vec{\mu}\cdot\vec{\mathcal{H}}$

1) topl. kapacitet

2) S i fazni vol.

3) Jeans – Rayl. zakon

$d'A = -\mathcal{H}\,dM$

$P \to -\mathcal{H}$

$V \to M$

$\Rightarrow C_{\mathcal{H}} - C_M = -T\left(\frac{\partial \mathcal{H}}{\partial T}\right)_M \left(\frac{\partial M}{\partial T}\right)_{\mathcal{H}}$

1) toplinski kapacitet

$C_P = \left(\frac{d'Q}{dT}\right)_P \qquad C_V = \left(\frac{d'Q}{dT}\right)_V$

$M = \chi \mathcal{H}$

$M = a\frac{\mathcal{H}}{T}$

$dU = d'Q + d'A$

opć. $d'A = \sum_i R_i \cdot dr_i$

spec. $d'A = P\,dV$

$dS \geq \frac{d'Q}{T}$ rev.

$dU = T\,dS + P\,dV$

$U(T,V)$

$dS = \frac{1}{T}dU + \frac{P}{T}dV \qquad dU = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV$

$\Rightarrow P + \left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V$

$d'Q = dU + d'A = d(\underbrace{U + PV}_{H})$

$P = \text{const}$

$\Rightarrow C_P = \left(\frac{\partial H}{\partial T}\right)_P$

$C_P = \left(\frac{\partial U}{\partial T}\right)_P + P\left(\frac{\partial V}{\partial T}\right)_P = \cancel{\left(\frac{\partial U}{\partial V}\right)_P \left(\frac{\partial P}{\partial T}\right)_U} + P\left(\frac{\partial V}{\partial T}\right)_P$

$C_P = \left(\frac{\partial U}{\partial T}\right)_V + \left(\frac{\partial U}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P + P\left(\frac{\partial V}{\partial T}\right)_P = C_V + \left[P + \left(\frac{\partial U}{\partial V}\right)_T\right]\left(\frac{\partial V}{\partial T}\right)_P$

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V \qquad Z = A \int_{\Gamma} e^{-\beta H} d\phi$$

$$U = -\left(\frac{\partial}{\partial \beta} \ln Z\right)_V$$

ideální plyn:

$N \approx \tilde{N}$

1-A $\quad U = \frac{3}{2} NKT \quad \Rightarrow \quad C_V = \frac{3}{2} NK = \frac{3}{2} R$

H_2 $80°K \rightarrow \frac{3}{2} KT$

2-A $\quad$ 3 tr. + 2 rot. $\quad \frac{3}{2} KT + \frac{2}{2} KT = \frac{5}{2} KT \quad \rightarrow C_V = \frac{5}{2} R$

oscil. $\quad \frac{3}{2} KT + \frac{2}{2} KT + KT = \frac{7}{2} KT$

Krystaly : $\quad U = 3NKT \quad$ Ko 3N harm. oscil.

$$C_V = 3NK \rightarrow \underline{\underline{3R}} \quad \Rightarrow \quad C_V \approx 6 \text{ cal/mol st.}$$

kovy: zanedbatelné st. slob. el. plynu

2) S i fazni volumen

$\theta = KT \qquad \beta = \frac{1}{\theta}$

$\Psi(\theta, a) = -\theta \ln Z$

$H = H(q_1, \dots p_s, a)$

$a = \{V, a_1, \dots a_k \dots\}$

$\downarrow$

$r_1 \quad \dots r_k$

$$\rho \equiv W = A\, e^{\frac{(\Psi(\theta,a) - H)}{\theta}}$$

$$1 = \int_\Gamma \rho\, d\phi = A \int_\Gamma e^{\frac{1}{\theta}[\Psi(\theta,a) - H]}\, d\phi$$

$$= A \cdot \frac{1}{Z} \int_\Gamma e^{-\frac{H}{\theta}}\, d\phi = A \cdot \frac{1}{Z} \int_{(H)} e^{-\frac{H}{\theta}} \frac{d\phi(H)}{dH}\, dH$$

$$\Omega(H, a) = \frac{d\phi(H)}{dH}$$

aprox: za $N \gg$ kan. $\to$ mikrok.

$$W(H) = A \cdot e^{+\frac{\Psi}{\theta}} \underbrace{e^{-\frac{H}{\theta}} \cdot \Omega(H, a)}_{P(H)} \to \delta(H - H_m)$$

$$\phi(H) = \int_{H \le H} d\phi \quad A e^{\frac{\Psi}{\theta}} e^{-\frac{H}{\theta}}$$

$$\int \rho \frac{d\phi}{dH}\, dH$$

$$\Rightarrow \overline{G(H)} = \int_{(H)} G(H) W(H)\, dH = G(H_m)$$

$$H_m = U_m = \bar{U}$$

$$1 = \int_\Gamma \rho\, d\phi = \rho(\bar{U}_m) \cdot \phi(\bar{U})$$

$$\Longleftrightarrow \quad Z = A e^{-\frac{\bar{U}}{\theta}} \cdot \phi(\bar{U})$$

$$R^2 = 2mH$$

$$\rho = A \frac{1}{Z} e^{-\frac{H}{\theta}}$$

projiceno za idealni 1-A plin gdje

$$\phi(H) = C R^{3N} = C \cdot (2mH)^{\frac{3N}{2}}$$

$$S = -\left(\frac{\partial F}{\partial T}\right)_V$$

$$\left.\begin{aligned} S &= -k \left(\frac{\partial \Psi}{\partial \theta}\right)_V \\ \bar{H} &= \Psi - \theta \left(\frac{\partial \Psi}{\partial \theta}\right)_V \end{aligned}\right\} \Rightarrow S = +k \frac{\bar{H} - \Psi}{\theta} = k \cdot \overline{\left(\frac{H - \Psi}{\theta}\right)}$$

$$A = \frac{1}{N! h^s} \qquad \frac{H - \Psi}{\theta} = -\ln\left(\rho \cdot \frac{1}{A}\right)$$

$$S = -k \cdot \overline{\ln \frac{\rho}{A}}$$

$$A = \frac{1}{N! h^s}$$

$$S = +k \overline{\ln \frac{A}{\rho}} = k \ln\left(A \frac{1}{\rho(\bar{U})}\right) \overset{\text{aprox.}}{=} \cancel{k \ln A \phi(U)}$$

$$= k \ln\left[A \cdot \frac{1}{\rho(\bar{U})}\right] \overset{\text{aprox}}{=} k \ln[A \phi(\bar{U})] = k \ln\left[\frac{1}{N!} \frac{\phi(\bar{U})}{h^s}\right]$$

3) Jeans-Rayl. zakon

$$\vec{A} = \sum_{\vec{q}} \vec{A}_q \cdot e^{i\vec{q}\vec{r}}$$

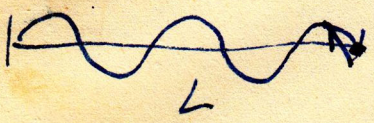
ekvival. ram. vala i harm. oscil.

$$H = \sum_q H_q$$

$$H_q = [P_q^2 + \omega^2(q) Q_q^2]$$

$$\overline{E(q)} = \overline{H_q} = kT$$

Kocka L^3



$$e^{iq_x \cdot x} = e^{iq_x(x+L)}$$

$$c = \lambda \cdot \nu = \lambda \cdot \frac{\omega}{2\pi}$$

$$i q_x \cdot L = i 2\pi \cdot n_x$$

$$q = \frac{2\pi}{\lambda} = 2\pi \cdot \frac{\omega}{2\pi} \cdot \frac{1}{c} = \frac{\omega}{c}$$

$$q_x = \frac{2\pi}{L} n_x$$

$$\Rightarrow d^3q = \left(\frac{2\pi}{L}\right)^3 d^3n$$

polariz.

$$U = 2 \sum_q \overline{E(q)}$$

$$\sum_i q_i \Rightarrow \int \frac{d\phi}{h^3}$$

$$\int \frac{d\phi}{h^3} = \frac{\int d^3\vec{r} \cdot \int d^3\vec{p}}{h^3} = \frac{V}{h^3} \int d^3p$$

$$p = \frac{h}{\lambda} = \frac{h}{2\pi} \cdot \frac{2\pi}{\lambda} = \frac{h}{2\pi} \cdot q \quad \Rightarrow d^3p = \left(\frac{h}{2\pi}\right)^3 d^3q$$

$$\Rightarrow \int \frac{d\phi}{h^3} = \frac{V}{h^3} \cdot \frac{h^3}{(2\pi)^3} \int d^3q = \frac{V}{(2\pi)^3} \int d^3q = \frac{V}{(2\pi)^3} \int \frac{1}{c^3} d^3\omega$$

$$= \frac{V}{(2\pi)^3 c^3} \cdot \int d^3\omega = \frac{V}{(2\pi)^3 c^3} \int_0^\infty (4\pi) \cdot \omega^2 d\omega$$

$\Rightarrow$

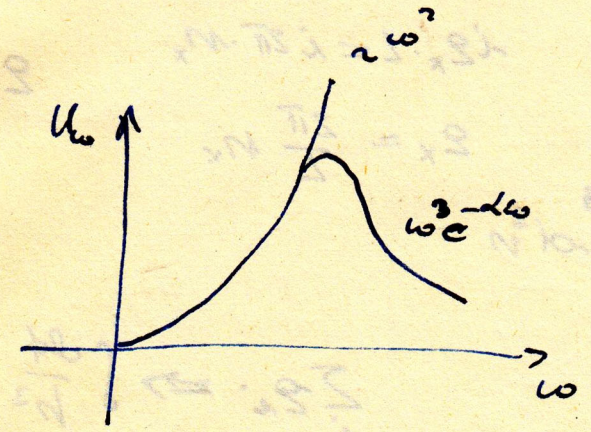
$$U = 2 \sum_{\varrho} \overline{E(\varrho)}$$

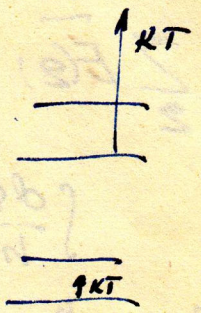
$$= 2 \cdot \int_0^\infty \overline{E(\varrho)} \cdot \frac{V \omega^2}{2\pi^2 c^3} d\omega$$

$$= \int_0^\infty V \cdot \frac{KT\omega^2}{\pi^2 c^3} d\omega$$

$$\frac{U}{V} = u = \int_0^\infty \underbrace{\frac{KT\omega^2}{\pi^2 c^3}}_{u_\omega} d\omega \quad \to \infty$$

$$u_\omega = \frac{KT\omega^2}{\pi^2 c^3}$$





slaganje za $\hbar\omega \ll KT$

ne slaže za $\hbar\omega \gg KT$

za visoke frekvencije

$\rho =$

$$\cancel{\rho = A \cdot \frac{1}{Z} e^{-3H}}$$

$$1 = \int_H A \cdot \frac{1}{Z} e^{-3H} \frac{d\phi(H)}{dH} dH$$

$$1 = \int \rho \, d\phi = \rho(\bar{u}) \phi(\bar{u}) = A \cdot \frac{1}{Z} e^{-3\bar{u}} \phi(\bar{u})$$

$$\underline{Z} = A \int_H e^{-3H} \frac{d\phi(H)}{dH} dH = A e^{-3\bar{u}} \cdot \phi(\bar{u}) \quad +$$

$$\rho = A \, \delta(H - H_m)$$

1) Čestica se nalazi u potencijalnoj jami beskonačno visokih zidova širine a, a njeno stanje opisano je valnom funkcijom $u(x) = Ax^2(a-x)$. Odredite vjerojatnost da se čestica nalazi u n-tom energetskom nivou

2) Odredite varijacionom metodom energiju osnovnog stanja harmonijskog oscilatora ako je probna funkcija oblika

$$u(x) = \frac{1}{\alpha^2 + x^2}$$

3) Procijenite pomoću relacije neodređenosti energiju osnovnog stanja čestice sa hamiltonijanom

$$H = \frac{p^2}{2m} + m\omega^2 r^2 - \frac{1}{16}\frac{\hbar^2}{m}\cdot\frac{1}{r^2}$$

4) Odredite ovisnost kemijskog potencijala, ukupne energije o temperaturi degeneriranog Fermijevog plina ako je gustoća stanja $D(E) = CE^2$

(zanemariti razvoj radijati na prva dva člana)

5) Izračunati koeficijent refleksije čestice koja nailazi na potencijalni barijeru oblika $u(x) = \begin{cases} 0 & x<0 \\ u_0 & 0<x\le b \\ 0 & a<x<b \\ \infty & x>b \end{cases}$

pri čemu vrijedi da je

$$E = \frac{\hbar^2\pi^2}{(b-a)^2}\cdot\frac{1}{2m}.$$

6) Izračunati koeficijent transmisije čestice koja nailazi na potencijalnu barijeru

$$u(x) = \begin{cases} u_0\frac{x}{a}\left(1-\frac{x}{a}\right) & 0\le x<a \\ 0 & \text{inače} \end{cases}$$

$$E = \frac{1}{8}u_0 \qquad u_0 = \frac{\hbar^2}{2ma^2}$$

7) Izračunati dijagonalne elemente izraza $x^2p^2 - p^2x^2$ za harmonijski oscilator

8) Kolika je kvadrat angularnog momenta u problemu kružnog simetrično simetrične potencijalne jame ako je $R(\rho) = \frac{m\rho\cos\rho}{\rho^2}$ $\rho = kr$ $E = \frac{\hbar^2k^2}{2m}$

rješ. $M^2 = -\hbar^2\left[2 + 2k^2r^2 - \frac{2mZe^2r}{\hbar^2}\right]$ za $r_0 = \frac{\hbar^2}{me^2}$

$$\rho = \frac{2Zr}{na_0}$$

$$\underline{M^2 = -\hbar^2\left[2 + 2\rho^2 - n\rho\right]}$$

9) Koeficijent refl. čestice na potencijalu

$$U(x) = \begin{cases} -30\,eV & x \leq 0 \\ 0\,eV & x > 0 \end{cases}$$

iznosi $R = 1/9$ odredite $E = ?$

10) Odredite M^2 i M_z za česticu opisanu valnom funkcijom

$$\psi(x,y,z) = A(x^2+y^2-2z^2)\,e^{-\frac{x^2+y^2+z^2}{2a^2}}$$

11) Izračunati varijacionom metodom energiju osnovnog stanja atoma vodika ako je probna funkcija oblika $u(\vec{r}) = \rho^2 e^{-\alpha\rho}$ $\rho = \frac{r}{a_0}$ $a_0 = \frac{\hbar^2}{me^2}$

12) Odredite kemijski potencijal nulnu energiju Fermijevog plina sastavljenog od N čestica ako je gustoća stanja $D(E) = DE^{3/2}$. Upotrebiti Sommerfeld razvoj do kvadratnog člana

μ_0 – kemijski potencijal na $T=0$

nije: $u = \frac{2}{5} D\mu^{5/2}\left(1 + \frac{35}{8}\left(\frac{kT}{\mu_0}\right)^2\pi^2\right)$ $\mu = \mu_0\left(1 - \frac{2}{3}\pi^2\left(\frac{kT}{\mu_0}\right)^2\right)$

13) Polazeći od Fermijeve raspodjele izračunati broj elektrona energije x komponenta brzine leži između v_x i $v_x + dv_x$ nije. $N = \frac{\sqrt{2}\, m^{3/2}}{4\hbar^3\pi^{3/2}}(kT)^{1/2} \cdot e^{(3\mu-2E)/kT}$

14) Stanje ravnotežnog rotatora u $t=0$ opisano je valnom funkcijom $\psi(\varphi,0) = A\cos^2 2\varphi$. Odredite valnu funkciju $\psi(\varphi,t)$

$$u_n = \frac{1}{\sqrt{2\pi}} e^{2n\varphi}$$ a odgovarajuće spektar energija $E_n = \frac{\hbar^2 n^2}{2J}$

rješenje: $\psi(\varphi,t) = \frac{1}{\sqrt{3\pi}}\left[1 + (2\cos^2 2\varphi - 1)e^{-i\frac{8\hbar}{J}t}\right]$

15) Čestica se nalazi u stanju danom valnom funkcijom $u(x,y,z) = A(x^2+y^2-z^2)e^{-\frac{x^2+y^2+z^2}{2a^2}}$

Odredite ΔM_x → standardnu devijaciju

16) Varijacionom metodom odredite energiju osnovnog stanja čestice koja se giba u potencijalu $U(r) = A/r^2 + Br^2$

ako je prolno funkcija $\phi(r) = c\, e^{-\alpha r}$

17) Odredite gustoću stanja sistema ako je energija

$E = s \cdot k^m$ s – spin k valni vektor

18) Spinska susceptibilnost degenerisanog elektronskog plina data je izrazom

$$\chi = -\mu_B^2 \int_0^\infty D(E) \frac{\partial f}{\partial E}\, dE$$

$$D(E) = \sqrt{\frac{2m^3 E}{\pi^2 \hbar^3}}$$

f – distribuciona funkcija elektrona

Izračunajte Sommerfeld. razvojem ovisnost susceptibilnosti o temperaturi do člana T^2.

Treba dobiti: $\chi = \chi_0 \left[1 + c \left(\frac{kT}{\mu_0}\right)^2\right]$

Treba naći χ_0 (susceptib. za $T=0$) i naći c

$\mu = \Gamma(T)$

kemijski potencijal

Zadání VAM – II dío

(1) Koeficient reflexe částice na potenciálové energii obdélníkové

$$U(x) = \begin{cases} -30\ eV & \text{za } x < 0 \\ 0\ eV & \text{za } x > 0 \end{cases}$$

18

Tvrdí $R = \frac{1}{9}$. Odhadněte $E = ?$

(2) Odhadněte H^2 i H_z za částice popisovaní vlnovou funkcí

$$\psi(x,y,z) = A(x^2+y^2-2z^2)\, e^{-\frac{x^2+y^2+z^2}{2a^2}}$$

(3) Zpřesňujte variačními metodami energii základní stavu atomu vodíku ale je probná funkce obdélníkové $U(r) = \rho^2 e^{-\alpha\rho}$ kde je $\rho = \frac{r}{a_0}$ $a_0 = \frac{\hbar^2}{me^2}$

(4) Odhadněte kumulativní potenciál, nebudete energie Fermiovy plyn sestavený od N částic, ale je konstantní stavy $D(E)$

$D(E) = DE^{7/2}$. Napomeno upotřebíte Sommerfeldův vzorec. Zadíváte se na závislosti do T^2 tj. $\left(\frac{kT}{\mu_0}\right)^2$ μ_0 – kumulativní potenciál na $T=0$

(5) Položíme od Fermiovy rozpadylé závislosti hustoty elektronů hybnosti x komponenta hybnosti leží v rozmezí v_x, $v_x + dv_x$

(6) Pokazati da za idealan plin izvan zoosnosti o statistici mjedi izraz $p = \frac{2}{3}\frac{E_{kin}}{V}$

(7) Klasičnim sistemu u dodiru s termostatom. Pokazati:

a) za srednji E_k na 1 st. slobode $1/2\, kT$

b) Zatim pokažite raspodjelu energije:

(8) Kristal. Debyeova aproksimacija daje za broj oscilacija $g(\omega)$

$$g(\omega) = \begin{cases} \frac{V}{2\pi^2}\left(\frac{1}{c_t^3} + \frac{2}{c_l^3}\right)\omega^2 = \frac{9N}{\omega_D^3}\omega^2 & \omega < \omega_D \\ 0 & \omega > \omega_D \end{cases}$$

Debyeova broj ω_D iz uvjeta

$$\int_0^{\omega_D} g(\omega)\, d\omega = 3N$$

Izračunati specif. toplinu za visoke i niske temperature.

9) Pokazati da za dovoljno visoke temperature molar. specif. toplina idealnog Fermijevog plina:

$$C_V = \frac{1}{3}\pi^2 k^2 T D(\mu_0)$$

10) Idealni Bozeov plin, N čestica u volumenu V. Neka je N_0 broj čestica u najnižem energetskom stanju ($\vec{p}=0$) a N' broj čestica u višem energetskim stanjima ($\vec{p}\neq 0$). Pokazati da pri temperaturama ispod neke kritične T_c, veličina N_0 postaje jednaka N i da je kemijski potencijal μ u toj oblasti 0.

11) Pokazati da je kemijski potencijal fotonskog plina 0 $N \neq$ const.

12) Kolikom broj $\bar{\varepsilon}$ u nekom uzorku je N, a gustoća stanja oblika

$$D = \text{const} \quad \text{za } \varepsilon > 0$$
$$D = 0 \quad \text{za } \varepsilon < 0$$

Naći

a) Naći fermijevu energiju μ_0 na $0°K$

b) izvesti izraz za degenerirani sistem

c) Pokazati da u slučaju velike degeneracije $C_V \sim T$

13) Pokazati da spinska magnetska ~~susceptibilnost~~ susceptibilnost sistema elektrona - pri proizvoljnoj temperaturi ima oblik

$$\chi = 2\mu_B^2 \int_0^\infty D'(\varepsilon) f(\varepsilon)\, d\varepsilon$$

14) Pokazati da je unutarnja energija idealnog Bozeovog plina u slučaju slabe degeneracije

$$E = \frac{3}{2} kTV \left(\frac{2\pi m kT}{h^2}\right)^{3/2} \sum_{s=1}^{\infty} \frac{e^{\frac{k\mu}{kT}}}{e^{5/2}}$$

15) Odrediti C_V i paramagnetsku susceptibilnost za degenerirani slobodni elektronski plin.

16) Położyć do rz. za fermiony w rozwiązaniu wyjątkowym może wystąpić. Boltzmanna rozpadnie.

17) Określić ułamek energii elektronowej plazmy w metalu na temp. T.

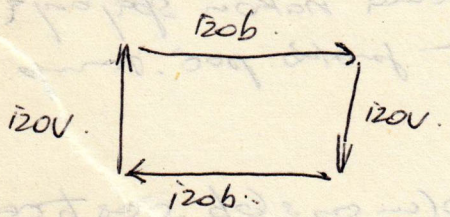
18) Znaleźć Kempfer potencjał i specif. ciepło silnie degeneracyjny ultrawiolotny ferm. plazmy ($E = cp$) rozważyć od centrum spina 1/2.

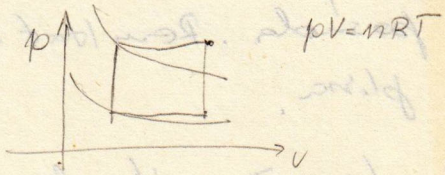
19) Znaleźć fluktuacje średnie liczbę ΔN_r fermionów plazmy w $T = 0°K$.

I dio zadaci

10.VII 72.

① Izračunati Q, W u ovakvom kružnom procesu





② Naći fazni volumen harm. oscilatora

③ Izrač. $\overline{\frac{1}{V^2}} \cdot \frac{1}{\sqrt{\overline{V^n}}}$ (Maxw. raspodjela)

rezul: $\frac{n}{\varkappa T}\left(1-\frac{1}{\sqrt{15}}\right)$

④ N neinteragirajućih čestica nalazi se u volum. V temp. T. Naći entalpiju tog sistema ako je pot. energ. čestice φ.

rez: $\frac{5}{2}NKT + N\varphi + NKT$

⑤ Plin sastavljen od n čestica zatvoren je u dvije posude volum. V_1 i V_2. Čestice u prvoj posudi imaju $E_p = \phi_1$, u drugoj $E_p = \phi_2$. Izrač. P plina u obje posude i ukupnu energ. sistema.

9.VI 72.

(1) Dva jednaka idealna plina sistemi broja čestica, tlakom nalaze se na raznim temp. u dvije posude raznih volumena. Izračunati promjenu S takvog sistema nakon spajanja posuda. Rezultat izraziti preko poč. temp. plina.

(2) Izračunajte fazni volumen sistema čestice sa masom jednakom masi elektrona u zatvorenom volumenu $V = 3cm^3$ i sa energijom

$$0{,}45 \leq E \leq 0{,}150\ eV$$

Rezultat izraziti u jedinicama h^s

$$m_e = 0{,}911 \cdot 10^{-27}$$

$$h = 6{,}625 \cdot 10^{-27}\ erg\,s$$

$$1eV = 1{,}6 \cdot 10^{-12}\ erg$$

(3) Odredite vrijednost izraza $\overline{V_r} \cdot \overline{\frac{1}{V_r}}$ za idealni plin gdje je $V_r = (\bar{V} - \bar{V}')$

(4) Naći F & C_V sistema $H = \frac{p^2}{2m} + ax^2 + bx$

(5) Plin sastavljen od n čestica zatvoren je u dvije posude volumena V_1 & V_2. Čestice u prvoj posudi imaju $E_p = \phi_1$, a u drugoj ϕ_2. Posude su spojene cijevima zanemarivog volumena. Izračunati tlak plina u obje posude i ukupnu energiju sistema

26.VI 72.

(1) Idealni plin expandira na dvostruki volumen. Izračunati promjenu S, Q i vanjski rad za

a) izotermni

b) izobarni proces

(2) Izračunati broj molek. id. plina koji u jedinici vremena udaraju u poprečnu posudu S' i imaju brzinu veću od srednje

(3) ? $\frac{2}{3}$ čestica od N ima energ. $4eV$, a $\frac{1}{3}$ čestica od N ima $2eV$. Kolika je temp. sistema

(4) $$H = \frac{p_x^2+p_y^2}{2m} + \alpha(x^2+y^2) + \beta xy$$

izrač. S & F

(5) ? N čestica u 2 volumena V_1 & V_2 spojena sa cijevi zanemarivog volumena imaju u V_1 potenc. energ. ϕ_1, i u V_2 ϕ. Izračunati kolik. je P & S sistema

(1) Dva jednaka idealna plina s istim brojem čestica i istom temp. nalaze se u dvije posude pri raznim tlakovima. Izračunati promjenu S' prilikom spajanja posuda.

(2) Za idealni plin sastavljen od N čestica izračunati Boltzman-ov funkcional

$$H = \int d^3\vec{r}\, f(\vec{r}) \log f(r)$$

(nema teksta do kraja)

(3) Odrediti broj čestica za Maxw. raspodjelu brzina između V_w i $2V_w$

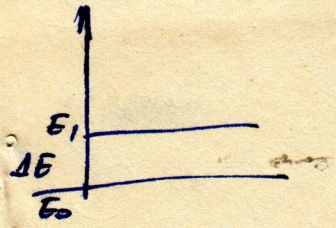
(4) Naći razliku term. kapaciteta čestice koja se giba u pot.

$$U(x) = \alpha x^2$$

i čestice koja se giba u pot. $U(x) = \beta |x|$

(5) Dan je sistem sa Hamiltonijanom

$$H = \frac{p_x^2 + p_y^2}{2m} + \alpha(x^2 + y^2) + \beta xy$$



$\Delta E \to 0$ kontin. energ. nivoi

valna fja. koja repr. stanje sistema, stanje može biti reprez. i sa E.

(Broj raspl. val. fja. odredjuje koliko je kv. broja)

$\hat{H} u = E u$ Šred. jba. u - valna fja

Somerfeld & Wilson: pravila kvantizacije

$$\oint p_k \, dq_k = n_k h \qquad n \in N \quad \Rightarrow \Delta\phi = h$$

$$\overline{G} = \frac{G_i n_i}{N} \qquad \overline{G} = \int G \, dW$$

$\int$

$$N_i = g_i \cdot e^{-1-\alpha} \cdot e^{-\beta E_i}$$

$$2 \cdot 10^{22} = e^{-1-\alpha} \cdot 1 \cdot 1$$

$$4 \cdot 10^{22} = e^{-1-\alpha} \cdot 1 \cdot e^{-\beta \cdot 1eV}$$

$$e^{-\beta \cdot 1eV} = \frac{4 \cdot 10^{22}}{2 \cdot 10^{22}}$$

$$e^{-\beta \cdot 1eV} = 2$$

$$Z = \cancel{\frac{1}{1-e}} =$$

$$N_i = g_i \cdot e^{-1-\alpha-\beta E_i} \qquad \beta = \frac{1}{kT} \qquad \underline{F = -kT \ln Z}$$

$$N = \sum_{i=0}^{2} N_i = \sum_{i=0}^{2} g_i e^{-1-\alpha} \cdot e^{-\beta E_i} = e^{-1-\alpha} \cdot \underbrace{\sum_{i=0}^{2} g_i \cdot e^{-\beta E_i}}_{Z}$$

$$W_i = \frac{N_i}{N} = \frac{g_i e^{-1-\alpha} e^{-\beta E_i}}{\sum_{i=0}^{2} g_i \cdot e^{-1-\alpha} e^{-\beta E_i}} \qquad i = 0,1,2$$

$$\underline{Z = \sum_{i=0}^{2} g_i e^{-\beta E_i}} \qquad N = e^{-1-\alpha} \cdot Z$$

$\cancel{ZA}$

Z

$$U = \sum_i E_i N_i \qquad U = -\left(\frac{\partial}{\partial \beta} \ln Z\right)_V$$

$$F = -kT \ln Z = -\frac{1}{\beta} \ln Z \qquad U = \frac{\partial}{\partial \beta}(\beta F)_V$$

$$U = \sum_{i=0}^{1} N_i E_i = U = (N_0 + N_1)\bar{E}$$

$$\bar{E} = \frac{\sum_{i=0}^{2} E_i e^{-\beta E_i}}{\sum_{i=0}^{2} e^{-\beta E_i}}$$

$$F = -\frac{1}{\beta} \ln Z \qquad Z = \sum_{i=0}^{2} e^{-\beta E_i}$$

1

$$N_i = e^{-1-\alpha} g_i e^{-\beta E_i}$$

$$\to e^{-1-\alpha} \int \frac{d\phi}{h^s} e^{-\beta E}$$

$$\sum N_i = N$$

$$W_i = \frac{N_i}{N}$$

$$e^{-1-\alpha} \sum_i g_i e^{-\beta E_i} = N$$

$$F = -kT \ln Z$$

$$\Rightarrow N_i = N \cdot \frac{g_i e^{-\beta E_i}}{\sum_i g_i e^{-\beta E_i}}$$

$$F = -NkT \ln Z$$

$$W_i = \frac{g_i e^{-\beta E_i}}{\sum_i g_i e^{-\beta E_i}}$$

$$N_i \to dN$$

$$U = -\frac{d}{d\beta}\left(N \ln Z\right)$$

$$dW = \frac{N_i}{N} \to \frac{dN}{N} = \frac{e^{-1-\alpha}}{N h^s} e^{-\beta E} d\phi$$

$$U = -\frac{d}{d\beta}\left(\ln Z^N\right)$$

$$\rho(E) = \frac{e^{-1-\alpha}}{N h^s} e^{-\beta E}$$

$$e^{1+\alpha} = \frac{1}{N h^s} \int e^{-\beta E} d\phi$$

$$W_i \to dW = \frac{e^{-\beta E} \frac{d\phi}{h^s}}{\int e^{-\beta E} \frac{d\phi}{h^s}}$$

$$Z = Z^N$$

$$e^{1+\alpha} = \frac{V}{N h^s} (2\pi m N T)^{3/2}$$

$$\frac{N_i}{g_i} = \frac{N h^3}{V (2\pi m k T)^{3/2}} e^{-\beta E_i} \ll 1$$

$$Z = \frac{1}{N h^s} \int e^{-\beta H} d\phi$$

$$Z = \sum_i g_i e^{-\beta E_i} \to \int \frac{d\phi}{h^s} e^{-\beta E}$$

$$\boxed{U = \sum_i E_i N_i} \qquad \sum_i N_i = N \qquad W_i = \frac{N_i}{N} = \frac{N_i}{\sum_i N_i} = \frac{e^{-1-\alpha} g_i e^{-\beta E_i}}{\sum_i e^{-1-\alpha} g_i e^{-\beta E_i}}$$

$$\frac{U}{N} = \sum_i E_i \frac{N_i}{N} = \sum_i E_i W_i = \frac{\sum_i E_i g_i e^{-\beta E_i}}{\sum_i g_i e^{-\beta E_i}} = -\frac{d}{d\beta} \ln\left(\sum_i g_i e^{-\beta E_i}\right)$$

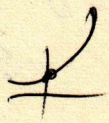
$$\frac{U}{N} = -\frac{d}{d\beta} \ln Z \Rightarrow$$

$e^{-\frac{\Delta S}{k}} \leq 1$ $\quad e^{\frac{\Delta S}{k}} \geq 1$ $\quad \frac{\Delta S}{k} > 0$ $\quad \underline{\underline{\Delta S > 0}}$

<u>Флуктуация</u>

$\frac{1}{e} \leq \frac{B}{B_m} \leq 1$ $\quad \frac{1}{e} \leq -\frac{\Delta S}{k}$

$\underline{e^{-1} \leq e}$



$\frac{1}{e} \leq \frac{1}{e^{\frac{\Delta S}{k}}}$

$e \geq e^{\frac{\Delta S}{k}}$ $\quad 1 \geq \frac{\Delta S}{k}$

$\delta U \neq 0 \Leftrightarrow \sum_i E_i \cdot \delta N_i \neq 0$

$\frac{d\rho}{dt} = 0$ $\quad \frac{d\Psi}{dt} = \frac{\partial \Psi}{\partial t}$

$\frac{\partial \rho}{\partial t} = 0$ t-din. ran. $\Rightarrow \frac{\partial \Psi}{\partial t} = 0 \Rightarrow \frac{d\Psi}{dt} = 0$ const. pba

$\Rightarrow \underline{\rho = \varphi(\mathcal{H})}$ Gibbs-ov postulat

$\underline{W = \frac{1}{\Omega(E,\mu)} \delta(H - E)}$ $\quad \underline{\int_\Gamma w \, d\Gamma = 1}$

$W(F) = \int_{(x)} \delta(F - F(x_1 \ldots x_n))\, w(x_1 \ldots x_n)\, dx_1 \ldots dx_n$

$\Psi = -\theta \ln Z = -kT \ln Z$

$W = A \frac{1}{Z} e^{-\beta H}$ $\quad A = \frac{1}{N! h^s}$ $\quad \underline{W = A\, e^{\frac{1}{\theta}(\Psi - H)}}$

$$W(F) = \int_{(x)} \delta(F - \underline{F(x_1 \ldots x_n)})\, w(x_1 \ldots x_n)\, dx_1 \ldots dx_n$$

$F = E \qquad \delta(F(x_1 \ldots x_n) - E)$

$$\bar{F} = \int_{(F)} F\, W(F)\, dF = \int_{(F)} F \left(\int_{(x)} \delta(F(x_1 \ldots x_n) - F)\, w(x_1 \ldots x_n)\, dx_1 \ldots dx_n \right) dF$$

$$= \int_{(x)} w(x_1 \ldots x_n) \left(\underbrace{\int_{(F)} F\, \delta(F - F(x_1 \ldots x_n))\, dF}_{F = F(x_1 \ldots x_n)} \right) dx_1 \ldots dx_n$$

$$= \int_{(x)} F(x_1 \ldots x_n)\, w(x_1 \ldots x_n)\, dx_1 \ldots dx_n$$

$$W(E) = \int_{\Gamma} \delta(E - H(q_1 \ldots p_s, a))\, w(q_1 \ldots p_s, a)\, d\phi$$

$$= \underline{A\, e^{\frac{\Psi - E}{\theta}} \cdot \Omega(E, a)} \qquad \Rightarrow \underline{Z = A \int_{(E)} e^{-\frac{E}{\theta}} \Omega(E, a)\, dE}$$

$$\underline{\bar{E} = \int E\, w(E)\, dE} = A\, e^{\frac{\Psi}{\theta}} \int_{(E)} e^{-\frac{E}{\theta}} \Omega(E, a)\, dE$$

$$= A \frac{1}{Z} e^{-\frac{E}{\theta}} \Omega(E, a)$$

$$Z = A \int_{(E)} e^{-\beta E} \cdot \Omega(E,a)\, dE \qquad \beta = \frac{1}{\theta} \qquad \frac{\partial}{\partial \beta} = \frac{\partial}{\partial \theta} \cdot -\frac{1}{\beta^2}$$

$$= -\theta^2 \frac{\partial}{\partial \theta}$$

$$\frac{\partial Z}{\partial \beta} =$$

$$\underbrace{U = Q - P dV}_{T ds} \qquad T = \left(\frac{\partial U}{\partial S}\right)$$

$$P = \left(\frac{\partial U}{\partial V}\right) \qquad P = -\left(\frac{\partial F}{\partial V}\right)_T$$

$$R_i = -\frac{\partial H}{\partial r_i} \qquad \overline{R_i} = -\left(\frac{\partial \varphi}{\partial r_i}\right)_{\theta, r_{i+1}, \ldots}$$

$$\underline{A_k = -\frac{\partial H}{\partial a_k}}$$

$$dA' = -E\, d\beta \qquad dA' = -\mathcal{H}\, dM \qquad M = \chi H \text{ dia} \qquad \mu = a\frac{\mathcal{H}}{T} \text{ para}$$

$$\bar{H} = -\left(\frac{\partial \ln Z}{\partial \beta}\right) = -\left(\frac{\partial \left(-\frac{1}{kT}\right)(-kT \ln Z)}{\partial \beta}\right) \qquad \varphi = -kT \ln Z$$

$$F = U - TS \qquad S = -\left(\frac{\partial F}{\partial T}\right)$$

$$U = F + TS$$

$$\underline{dH + \sum_k A_k da_k} \qquad dF = TdS - PdV - TdS - SdT$$

$$\theta \qquad = SdT - PdV$$

$$\theta = kT \qquad -\left(\frac{\partial \varphi}{\partial r_i}\right)_{\theta, \ldots}$$

$$\underline{S' = -\left(\frac{\partial \varphi}{\partial T}\right)_a}$$

$$dA' = [dH]_{[a]} = -\sum_k \frac{\partial H}{\partial a_k} da_k$$

$$dU = dQ' + d'A \qquad A_k = -\frac{\partial H}{\partial a_k} \qquad R_i = -\frac{\partial H}{\partial r_i}$$

$$E_p(x_1 \dots x_N) = E_p(0) + \sum_i \left(\frac{\partial E_p}{\partial x_i}\right)_0^{=0} x_i + \frac{1}{2}\sum_i^N \sum_j^N \left(\frac{\partial^2 E_p}{\partial x_i \partial x_j}\right)_0 x_i x_j$$

$$\sum_{i=1}^{3N} x_i \frac{\partial H}{\partial x_i} = \sum_{i=1}^{3N} x_i \frac{\partial E_p}{\partial x_i} \qquad \frac{\partial E_p}{\partial x_i} = \sum_j (\quad) x_j = 2(E_p - E_p(0))$$

$$= \sum_i^N \sum_j^N \left(\frac{\partial E_p}{\partial x_i x_j}\right)_0 x_i x_j = 2(E_p - E_p(0))$$

$$\frac{1}{2} \cdot 3NkT + \frac{1}{2} N 3 kT = 3NkT$$

SJ H H_2 50°K

2-A $\frac{3}{2}kT + 2 \cdot \frac{kT}{2} = \frac{5}{2}kT \rightarrow \frac{3}{2}kT$

$$\frac{3}{2}kT + \frac{2}{2}kT + kT = \frac{7}{2}kT$$

NK

V is A $\frac{3}{2}kT + \frac{3}{2}kT$ rot.

~~n-atomic~~ molek.

n-atomic

3n - st. stos.

(3n-6) oscil.

$$(3n-6)kT + 3 \cdot \frac{kT}{2} + 3 \cdot \frac{kT}{2}$$

na prawa 3 transl. + 2 rot. + (3n-5) · kT

$$\rho \sim e^{-\frac{E}{kT}}$$

$$Z = A\int e^{-\beta H}\,dp$$

$$Z_p = a\int_0^l e^{-\frac{mgx}{kT}}\,dx = \frac{akT}{mg}\left(1-e^{-\frac{mgl}{kT}}\right)$$

$$Z_p^N$$

$mgl \gg kT$

$$E_p = -\frac{d}{d\beta}(\ln Z_p) = \boxed{kT}$$

$mgl \ll kT$

$$\bar{E}_p = \frac{mgl}{2} \ll kT$$

$$U = aVT^4 \qquad E = c\sqrt{p^2+m_0^2c^2} \qquad E = h\cdot\nu$$

$$E = c\cdot p \qquad p = \frac{h}{\lambda} \leftarrow \frac{h}{c}\cdot\nu$$

$$\vec{E} = -\frac{1}{c}\dot{\vec{A}} \qquad \vec{H} = \mathrm{rot}\,\vec{A} \qquad \mathrm{div}\,\vec{A} = 0$$

$$c = \lambda\cdot\nu$$

$$\boxed{\vec{A} = \sum_{\vec{q}} \vec{A}_q\, e^{i\vec{q}\cdot\vec{r}}} \qquad q = \frac{2\pi}{\lambda} \qquad \ddot{A}_q + \omega^2(q)A_q = 0$$

$$H_q = \frac{1}{2}\left[P_q^2 + \omega^2(q)Q_q^2\right] \Rightarrow \bar{E}_q = \bar{H}_q = kT$$

$$H = \sum_q H_q \qquad \boxed{0 < \omega(q) < \infty} \qquad \omega = 2\pi\nu = \frac{2\pi c}{\lambda} = c\cdot q$$

$$U = 2\sum_q \bar{E}(q) \qquad V = L^3 \qquad e^{iq_x\cdot x} = e^{iq_x(x+L)}$$

$$\Rightarrow iq_x\cdot L = i2\pi n_x \Rightarrow \boxed{q_x = \frac{2\pi}{L}\cdot n_x}$$

$L \to \infty$ q_x - квантовано

$$d^3q = \left(\frac{2\pi}{L}\right)^3 d^3n$$

(5)

$\vec{A} = \sum_{\vec{q}} \vec{A}_{\vec{q}} e^{i\vec{q}\vec{r}} \qquad \vec{H} = \sum_{\vec{q}} \vec{H}_{\vec{q}}$

$$d^3q = \left(\frac{2\pi}{L}\right)^3 d^3n$$

$$\sum_{\vec{q}} F(\vec{\varepsilon}) \to \int F(\varepsilon)\, d^3n = \frac{V}{(2\pi)^3}\int F(q)\, d^3q$$

$$p = \frac{h}{\lambda} = \frac{h}{2\pi}\cdot\frac{2\pi}{\lambda} = \frac{h}{2\pi}q$$

$$\Rightarrow d^3p = \left(\frac{h}{2\pi}\right)^3 d^3q = \left(\frac{h}{\lambda}\right)^3 d^3n$$

$$n_i \to \int \frac{d\phi}{h^3} = \int \frac{d^3\vec{r}\, d^3\vec{p}}{h^3} = \frac{V}{h^3}\int d^3p = \left(\frac{h}{2\pi}\right)^3 \frac{1}{h^3}\int d^3q = \frac{V}{h^3}\cdot\frac{h^3}{\lambda^3}\int d^3n$$

$$U = 2\sum_{q} \overline{E(q)} = 2\sum_i E_i(\varepsilon)\cdot n_i \to 2\int \overline{E(q)}\, d^3n$$

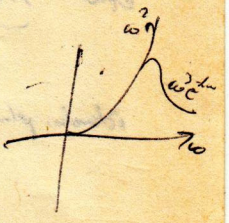
$\overline{E(q)} = \overline{\varepsilon} = kT$

$$U = 2\cdot\frac{V}{(2\pi)^3}\int \overline{E(q)}\, d^3q$$

$\omega = q\cdot c \qquad d^3q = \frac{1}{c^3}d^3\omega = \frac{4\pi}{c^3}\cdot\omega^2 d\omega$

$$U = 2\cdot\frac{4\pi V}{c^3(2\pi)^3}\int_0^\infty \overline{E(\varepsilon)}\,\omega^2 d\omega$$

$$U = \frac{VkT}{\pi^2 c^3}\int_0^\infty \omega^2 d\omega = \int_0^\infty \frac{VkT\omega^2}{\pi^2 c^3}\, d\omega$$



$$du = \frac{kT\omega^2}{\pi^2 c^3}\, d\omega \qquad \boxed{u_\omega = \frac{du}{d\omega} = \frac{kT\omega^2}{\pi^2 c^3}}$$

za $kT \gg \hbar\omega$ exp. sfanje

$$\sum_i q_i \to \int \frac{d\phi}{h^3} = \int \frac{d^3\vec{r}\, d^3\vec{p}}{h^3} = \frac{V}{h^3}\cdot\int d^3p = \frac{V}{h^3}\left(\frac{h}{2\pi}\right)^3\int d^3q = \frac{V}{(2\pi)^3}\int d^3q$$

$$U = 2\sum_q E(q) \to 2\cdot\frac{V}{(2\pi)^3}\int E(q)\, d^3q = \frac{2V\cdot kT}{(2\pi)^3}\int d^3q = 2kT\int d^3n$$

$$d^3q = \frac{d^3\omega}{c^3}$$

$$\Rightarrow u = \frac{U}{V} = \int \frac{2kT}{(2\pi)^3 c^3}\cdot 4\pi\omega^2 d\omega = \int_0^\infty \underbrace{\frac{kT\omega^2}{\pi^2 c^3}}_{u_\omega} d\omega \to \infty$$

$$F = -kT \ln Z$$

$$Z = A V^N (2\pi m k T)^{\frac{3N}{2}} \qquad S = -\left(\frac{\partial F}{\partial T}\right)_V$$

$$V = V_1 + V_2 \qquad N = N_1 + N_2$$

$$\Rightarrow A = \frac{1}{N!\, h^{3N}} \qquad \Leftrightarrow S_1 + S_2 = S$$

$$N_i = e^{-1-\alpha} q_i e^{-\beta E_i} \qquad \frac{\Delta q}{h^3} \Rightarrow \int \frac{d\phi}{h^3} \qquad \Omega(E) = \frac{e^{-1-\alpha}}{N h^3} e^{-\beta E} \frac{\frac{1}{h^3} e^{-\beta E}}{\frac{1}{h^3} \int e^{-\beta E} d\phi}$$

$$1 - A \Rightarrow 3N$$

$$\phi(H) = \int_{H \le U} d\phi \qquad g = A \frac{1}{Z} e^{-\beta H} \qquad \Omega(U, a) = \int \delta(U - H)\, d\phi$$

$$d\phi = \frac{d\phi(H)}{dH} dH \qquad d\phi(H) = \underbrace{\frac{\partial \phi(H)}{\partial H}}_{\Omega(H)} \cdot dH \qquad W(E) = \int \delta(E - H) w(H \ldots)\, d\phi$$

$$= \int \delta(H - E) \cdot A e^{\frac{E-H}{\theta}} d\phi$$

$$dW = \underbrace{A e^{\beta F} e^{-\beta H} \cdot \frac{d\phi(H)}{dH}}_{P(H)} dH \qquad = A e^{\frac{F-E}{\theta}} \cdot \underbrace{\int \delta(H - E) \delta_{u_1 = E} du_1 \cdot du_s}_{\Omega(E, a)}$$

ideální plyn $\quad \phi(H) \sim H^{\frac{3N}{2}} \qquad P(H) \to \delta(H - U_m)$

$$\Omega(H) \qquad P(H) = \frac{1}{A e^{\beta F}} W(H) \to \frac{1}{A e^{\beta F}} \delta(H - H_m)$$

$$\Rightarrow \overline{G(H)} = \int G(H) W(H)\, dH = G(H_m)$$

$$w = e^{-\beta H}$$

$$\int W(E) dE = 1$$

$$\overline{W} = \int w \cdot W(E) dE = W(\bar{U})$$

$$Z = A \int e^{-\beta H} d\phi = A \int_0^\infty e^{-\beta H} \underbrace{\frac{d\phi}{dH}}_{\Omega(H)} dH \overset{\text{aprox.}}{\approx} A e^{-\beta \bar{U}} \phi(\bar{U})$$

$$S = -\left(\frac{\partial F}{\partial T}\right)_V = \frac{U - \bar{H}}{T} \qquad U = -\left(\frac{\partial}{\partial \beta} \ln Z\right)_V \qquad F = -kT\left(\ln A\phi(U) - \beta \bar{U}\right)$$

$$U = F + TS$$

$$\bar{H} = U + T\left(\frac{\partial U}{\partial T}\right)_V \qquad S = -\left(\frac{\partial F}{\partial T}\right) = kT \ln\left(A\phi(\bar{U})\right) \qquad A\phi(\bar{U}) = \frac{1}{N!} \frac{\phi(\bar{U})}{h^s}$$

$$1 = \int w\, d\phi = W(\bar{U}) \phi(\bar{U})$$

$$w = \frac{1}{\Omega(E, a)} \delta(H - E)$$

$$S = -k \ln \frac{W}{A} = k \ln \frac{A}{W}$$

$$W \to \frac{1}{\Omega(E, a)} \delta(H - H_m)$$

$Z = A\int e^{-\beta E}\Omega(E)dE$

6

$$\phi(H) = \int_{H\le U} d\phi = V^N \cdot \int dp_1 \dots dp_{3N} \qquad H = \frac{1}{2m}(p_1^2 + \dots p_{3N}^2) = R^2$$

$$= CV^N \cdot R^{3N} = C\cdot V^N (2mH)^{\frac{3N}{2}}$$

$$P(H) = e^{-\beta H}\frac{d\phi}{dH} = \quad \Rightarrow H_m = \frac{3N}{2}kT \quad N\gg 1 \quad \Rightarrow \phi(\bar U) = CV^N(3NmkT)^{\frac{3N}{2}}$$

aprox: $Z \simeq Ae^{-\beta\bar U}\phi(\bar U) = ACV^N(3NmkT)^{3N/2}\cdot e^{-\beta\bar U} \cdot e^{-\frac{3N}{2}}$

$$\Rightarrow S = k\ln A\phi(\bar U) = k\ln\left(ACV^N(3NmkT)^{\frac{3N}{2}}\right)$$

exact. $Z = A\int e^{-\beta H}d\phi = A\int e^{-\beta H}\frac{d\phi(H)}{dH}dH \overset{N\text{-prom}}{=} ACV^N(2mkT)^{\frac{3N}{2}}\cdot\left(\frac{3N}{2}\right)!$

$$= Ae^{-\beta U}\cdot\phi(\bar U)$$

$F = -\theta\ln Z \qquad F(T,a) \qquad \frac{1}{\theta} = \beta$

$$1 = A\int e^{\beta(F-H)}d\phi \left(\frac{\partial}{\partial\beta}\right)_V$$

$$0 = A\int e^{\beta(F-H)}\Big[\underbrace{\frac{\partial}{\partial\beta}(\beta F)}_{\bar H} - H\Big]d\phi \Big/ \left(\frac{\partial}{\partial\beta}\right)_V \Rightarrow \bar U = \bar H = \left(\frac{\partial\beta F}{\partial\beta}\right)$$

$$\Rightarrow \frac{\Delta U^2}{\bar U} = \frac{\partial(\frac{1}{\bar U})}{\partial\beta} \sim \frac{1}{\sqrt{N}}$$

$$W(E) = \int_\Gamma \delta(H-E)W(p,q)d\phi = Ae^{\beta(F-E)}\cdot\Omega(E)$$

$$G = \sum_{i=1}^{W} g_i \qquad \Delta G^2 = \overline{G^2} - \bar G^2 \qquad \Rightarrow \frac{\Delta G}{\bar G} = \frac{\Delta g}{\bar g}\cdot\frac{1}{\sqrt{N}}$$

$$g = A\frac{1}{Z}e^{-\beta H}$$

$$\mathcal{H} = \overline{\ln g} = \int_\Gamma g\ln g\, d\phi \qquad \frac{d\mathcal{H}}{dt} = \int_\Gamma \frac{\partial g}{\partial t}(1+\ln g)d\phi$$

$1 = \int g\, d\phi$

$1 = g(\bar U)\phi(\bar U)$

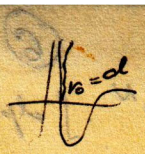
$$\frac{\partial g}{\partial t} = 0 \Rightarrow \frac{d\mathcal{H}}{dt} = 0 \qquad \Rightarrow \text{opc: } S' = -k(\underbrace{\ln W}_{\ln g} + \ln W'/b^s)$$

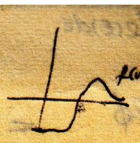
$$\boxed{Z = Ae^{-\beta U}\cdot\phi(\bar U)}$$

$$\Rightarrow \boxed{S' = -k\mathcal{H}} \qquad S = -k\overline{\ln g} \qquad \overline{\ln g} = \ln g(\bar U) \qquad g(\bar U) = \frac{1}{\phi(\bar U)}$$

$$S = k\ln\frac{A}{W} = k\ln\frac{A}{W(\bar U)} = k\ln A\phi(\bar U)$$

$$E_p(r) \simeq \frac{c_1}{r^{12}} - \frac{c_2}{r^6} \qquad c_1, c_2 > 0$$





$$f(r) = e^{-\frac{\beta E_p(r)}{}} - 1$$

$$Z_{int} = \frac{1}{V^N} \int \prod_{ij}^{C_N^2} (1 + f_{ij})\, d^3r_1 \cdots d^3r_N \qquad 2B(T) = \int (1 - e^{-\beta E_p(r)})\, d^3r$$

$$P = -\left(\frac{\partial F}{\partial V}\right)_{T,a} \qquad \overline{A_k} = -\left(\frac{\partial \psi}{\partial a_k}\right)_{\theta_1}$$

$$P = NkT\left(1 + B(T)\frac{N}{V}\right) \qquad \text{Boyl. temp} \qquad T_B$$

Kvantna statist. fizika

Boltzmann-ova - razlik. čestica

$N_i = g_i \cdot e^{-1-\alpha} \cdot e^{-\beta E_i}$

$$\sum_i N_i = N \qquad \sum_i N_i E_i = U$$

bozoni $\quad g_i \gg 1 \quad N_i \gg 1$

$$B_i = \frac{(N_i + g_i - 1)!}{(g_i - 1)!\, N_i!}$$

$$B = \prod_i \frac{(N_i + g_i)!}{g_i!\, N_i!} \qquad \delta(\ln B - \alpha N - \beta U) = 0$$

$$N_i = \frac{g_i}{e^{\alpha + \beta E_i} - 1}$$

Bose-ova raspodjela

fiz. smisao $\quad \varrho_i = \frac{N_i}{g_i} > 0 \Rightarrow e^{\alpha + \beta E_{min}} \geq 1$

izvan djelov. vanj. polja $\Rightarrow U = 0 \Rightarrow E = E_{kin.} \Rightarrow E_{min} = 0 \Rightarrow$

$$e^{\alpha} \geq 1$$

- formalno očuvano za $\alpha = 0$ $\quad$ tj. raspodj. modifiti $\sum N_i = N \;/ \cdot \alpha$

fotoni, fononi - broj nije očuvan

$\alpha = 0 \quad E_i = \hbar\omega \Rightarrow \varrho_i = \varrho(E_i)$

$\beta = \frac{1}{KT} \quad \Rightarrow \varrho(\omega) = \frac{1}{e^{\frac{\hbar\omega}{KT}} - 1}$ $\quad$ Planck-ova raspodjela

fermioni - Pauli-ev princip (e^-, ...

$$B = \prod_i \frac{g_i!}{N_i!\,(g_i - N_i)!} \qquad \delta(\ln B - \alpha N - \beta U) = 0$$

$$N_i = \frac{g_i}{e^{\alpha + \beta E_i} + 1}$$ Fermi-ova raspodj.

veza s klas. statistikom

za $e^{\alpha + \beta E_i} \gg 1 \Rightarrow g_i = e^{-\alpha} e^{-\beta E_i}$

Veza α i β s t-din. veličinama

$S = k \ln B$ - opć. izvedeno iz $\begin{cases} S = S_1 + S_2 \\ B = B_1 \cdot B_2 \end{cases}$

$$\Rightarrow \delta S = k\alpha \cdot \delta N + k \cdot \beta\, \delta U$$

$$k\alpha = \left(\frac{\partial S}{\partial N}\right)_{U,N} \qquad k\beta = \left(\frac{\partial S}{\partial U}\right)_{N,V}$$

T-dan. sistem $\quad V, N, T, P, U, S$ — ukoliko se menjaju broj čestica

$V = N \cdot v$ $\quad$ v - vol. na 1 česticu

$S = N \cdot s$

$U = N \cdot u$

$$\delta V = N \cdot \delta v + v \delta N$$

$$\Rightarrow \delta v = \frac{\delta V - v \delta N}{N}$$

$$\delta s = \frac{\delta S - s \delta N}{N}$$

$$\delta u = \frac{\delta U - u \delta N}{N}$$

Osnovna t-dn. relacija za 1 česticu glasi:

$$T ds = \delta u + P \delta v$$

$$T\delta S - Ts\delta N = \delta U - u\delta N + P\delta V - Pv\delta N$$

$$\Rightarrow T\delta S = \delta U + P\delta V - (u + Pv - Ts)\delta N$$

proširenje na sist. gde se menja broj čestica

$$G = U + PV - TS$$

po analogiji def: $\mu = u + Pv - Ts$ $\quad$ Hemijski potencijal za 1 česticu

$$\Rightarrow G = N\mu$$

$$\delta S = \frac{1}{T}\delta U + \frac{P}{T}\delta V - \frac{\mu}{T}\delta N$$

$$\Rightarrow \frac{1}{T} = \left(\frac{\partial S}{\partial U}\right)_{V,N} \qquad -\frac{\mu}{T} = \left(\frac{\partial S}{\partial N}\right)_{U,V}$$

$$\Rightarrow \alpha = -\frac{\mu}{KT} \qquad \beta = \frac{1}{KT}$$

isti su rezultati za sve tri statistike

$$\boxed{g_i = \frac{1}{e^{\frac{E_i-\mu}{KT}} \pm 1}}$$

$\rightarrow$ bozoni $e^{\alpha} \geq 1 \Rightarrow \alpha \geq 0 \Rightarrow \mu \leq 0$ kem. potencijal za bozone neg. ili nula

prijelaz kvant. stat. $\rightarrow$ klas.

za $U=0 \Rightarrow E_{min}=0 \Rightarrow e^{\alpha} \gg 1 \Rightarrow e^{-\beta\mu} \gg 1 \Rightarrow e^{\beta\mu} \ll 1$

T-din. ravnoteža

$\Rightarrow \delta S = 0$ za ravnot.

2 sistema

$$V = V_1 + V_2$$
$$N = N_1 + N_2$$
$$S = S_1 + S_2$$
$$U = U_1 + U_2$$

V, N, U konst.

$\Rightarrow \delta V_1 = -\delta V_2$

$$\delta S = \delta S_1 + \delta S_2 = \frac{\delta U_1 + P_1 \delta V_1 - \mu_1 \delta N_1}{T_1} + \frac{\delta U_2 + P_2 \delta V_2 - \mu_2 \delta N_2}{T_2} = 0$$

$$\delta U_1 \left(\frac{1}{T_1} - \frac{1}{T_2}\right) + \delta V_1 \left(\frac{P_1}{T_1} - \frac{P_2}{T_2}\right) - \delta N_1 \left(\frac{\mu_1}{T_1} - \frac{\mu_2}{T_2}\right)$$

zbog nezavisnosti U, N, V

$$\Rightarrow \left.\begin{array}{l} T_2 = T_1 \\ P_2 = P_1 \end{array}\right\} \text{odranyt}$$

$\mu_2 = \mu_1$ - bitan za razumij. rada tranzistora u fizici čvrstog stanja

Velikokanonski sistem

bozoni: našli smo

$$\ln B = \sum_i \left[(g_i + N_i) \ln (g_i + N_i) - g_i \ln g_i - N_i \ln N_i \right]$$
$$= \sum_i g_i \left[\ln (1 + \delta_i) + \delta_i \ln \frac{1 + \delta_i}{\delta_i} \right]$$

$$\delta_i = \frac{N_i}{g_i} = \frac{1}{e^x - 1} \qquad x = \frac{E_i - \mu}{kT}$$

$$\Rightarrow \ln B = -\sum_i g_i \ln\left[1 - e^{\partial - \beta E_i}\right] + \underbrace{\sum_i \frac{N_i E_i - \mu N_i}{kT}}_{\frac{U-\mu N}{kT} = \beta U - \mu\beta N}$$

$$\boxed{\nu = \beta \cdot \mu}$$

$$S' = k \ln B$$

$$\Omega = -\sum_i g_i \ln\left(1 - e^{\partial - \beta E_i}\right)$$

veličina Hamiltonova potenciál

$$\Rightarrow S' = k(\Omega + \beta U - \nu N)$$

$$\delta S' = k(\delta\Omega + \beta\delta U + U\delta\beta - \nu\delta N - N\delta\nu)$$

t-dim. zápis $\delta S' = k(\beta\delta U + \beta P\delta V - \beta\mu\delta N)$

$$\Rightarrow \delta\Omega = \beta P\delta V - U\delta\beta + N\delta\nu$$

$$\Rightarrow P = kT\left(\frac{\partial\Omega}{\partial V}\right)_{\beta,\nu} \quad U = -\left(\frac{\partial\Omega}{\partial\beta}\right)_{V,\nu} \quad N = \left(\frac{\partial\Omega}{\partial\nu}\right)_{V,\beta}$$

(*)

fermioni

analogno:

$$\Omega = +\sum_i g_i \ln\left(1+e^{\nu-\beta E_i}\right) \qquad U = -\left(\frac{\partial\Omega}{\partial\beta}\right)_{V,\nu} \quad (*)$$

$$N = \left(\frac{\partial\Omega}{\partial\nu}\right)_{\beta,V}$$

(*) sadrže sve prij. izraze kao spec. slučaj

specijalno:

fotoni & fononi

$$\alpha=0 \Rightarrow \mu=0 \quad \nu=0 \quad E_i=\hbar\omega$$

$$\Omega = -\sum_{\vec{k}\sigma} \ln\left(1-e^{\nu-\beta\hbar\omega}\right)_{\nu=0}$$

$$S = k\left(\Omega+\beta U-\nu N\right)_{\nu=0}$$

$$\Rightarrow \Omega = \frac{S}{k} - \frac{U}{kT} = -\frac{1}{kT}\left(\underbrace{U-TS}_{F}\right)$$

$$\Rightarrow F = -kT\,\Omega$$

$$F = kT\sum_{\vec{k}\sigma} \ln\left(1-e^{\nu-\beta\hbar\omega}\right)_{\nu=0}$$

osnovni izraz. kad proučavamo fot. i fon. sistema

$$\Omega = \mp \sum_i g_i \ln\left(1 \mp e^{\nu - \beta E_i}\right)$$

bozoni

Veza s klas. statistikom

$\nu = \frac{\mu}{kT} = \beta\mu$

tj. $U=0$ $E_{min}=0$ $\Rightarrow e^{\alpha} \gg 1$ $\Rightarrow e^{\beta\mu} \ll 1$

$\alpha = -\nu$

Klas. limes $\underline{e^{\nu} \ll 1}$ $\Rightarrow e^{\alpha} \gg 1$

ne za bozone $e^{\alpha - \beta E_i} \gg 1$

od velikokan. potenc.

$$x = \underline{e^{\nu - \beta E_i} \ll 1}$$

$$\ln(1 \mp x) \approx \mp\left[\pm x \pm \frac{x^2}{2}\right] \approx \mp x$$

$$\Omega = \sum_i g_i e^{\nu - \beta E_i}$$

$$= e^{\nu} \underbrace{\sum_i g_i \cdot e^{-\beta E_i}}_{Z \text{ - partic. fja. 1 čestice}}$$

$$N = \left(\frac{\partial \Omega}{\partial \nu}\right)_{V,\beta} \Rightarrow \underline{N = e^{\nu} \cdot Z} \Rightarrow e^{\nu}$$

novo:

$$U = -e^{\nu} \cdot \left(\frac{\partial Z}{\partial \beta}\right)_{\nu,V} = -N \left(\frac{\partial \ln Z}{\partial \beta}\right)_{\nu,V}$$

$Z = z^N$

$$\Rightarrow \underline{F = -kTN \ln z} \Rightarrow U = \left(\frac{\partial \beta F}{\partial \beta}\right)_{\nu,V}$$

$$P = kT e^{\nu} \left(\frac{\partial Z}{\partial V}\right)_{\beta,V} = NkT \left(\frac{\partial \ln z}{\partial V}\right)_{\beta,V} = -\left(\frac{\partial F}{\partial V}\right)_{\beta,V}$$

$e^{\nu} = \frac{N}{z}$

Maxw. relac.

$$A(0) = \begin{pmatrix} 0 & 0 \\ i & 0 \end{pmatrix} \qquad i = x_1(0) = C_1$$
$$0 = x_2(0) = C_2$$

$$A(t) = \begin{pmatrix} 0 & x_2(t) \\ x_1(t) & 0 \end{pmatrix} \Rightarrow \dot{A}(t) = \begin{pmatrix} 0 & 0 \\ i e^{-\frac{\varepsilon}{i\hbar}t} & 0 \end{pmatrix}$$

$$\dot{A} = \frac{1}{i\hbar}\left[AH - HA\right]$$

$$AH = \begin{pmatrix} 0 & x_2 \\ x_1 & 0 \end{pmatrix}\begin{pmatrix} \varepsilon & 0 \\ 0 & 2\varepsilon \end{pmatrix} = \begin{pmatrix} 0 & 2\varepsilon x_2 \\ \varepsilon x_1 & 0 \end{pmatrix}$$

$$HA = \begin{pmatrix} \varepsilon & 0 \\ 0 & 2\varepsilon \end{pmatrix}\begin{pmatrix} 0 & x_2 \\ x_1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & \varepsilon x_2 \\ 2\varepsilon x_1 & 0 \end{pmatrix}$$

$$\dot{A}(t) = \frac{1}{i\hbar}\left\{\begin{pmatrix} 0 & 2\varepsilon x_2 \\ \varepsilon x_1 & 0 \end{pmatrix} - \begin{pmatrix} 0 & \varepsilon x_2 \\ 2\varepsilon x_1 & 0 \end{pmatrix}\right\}$$

$$= \frac{1}{i\hbar}\begin{pmatrix} 0 & \underline{\underline{\varepsilon x_2}} \\ \underline{\underline{-\varepsilon x_1}} & 0 \end{pmatrix} = \begin{pmatrix} 0 & \frac{\varepsilon}{i\hbar} x_2(t) \\ -\frac{\varepsilon}{i\hbar} x_1 & 0 \end{pmatrix}$$

$$\Rightarrow \dot{x}_1(t) = -\frac{\varepsilon}{i\hbar} x_1 \qquad \dot{x}_2(t) = \frac{\varepsilon}{i\hbar} x_2(t)$$

$$\frac{\dot{x}_1(t)}{x_1} = -\frac{\varepsilon}{i\hbar} \qquad \frac{\dot{x}_2(t)}{x_2(t)} = \frac{\varepsilon}{i\hbar}$$

$$\frac{dx_1(t)}{x_1} = -\frac{\varepsilon}{i\hbar}dt \qquad \frac{dx_2(t)}{x_2(t)} = \frac{\varepsilon}{i\hbar}dt$$

$$x_1(t) = C_1 e^{-\frac{\varepsilon}{i\hbar}t} \qquad x_2(t) = C_2 e^{\frac{\varepsilon}{i\hbar}t}$$

(5)

$$U(r) = \frac{A}{r^2} - \frac{B}{r}$$

$$\frac{d^2R}{dr^2} + \frac{2}{r}\frac{dR}{dr} + \frac{2m}{\hbar^2}\left[E - \frac{\hbar^2}{2m}l(l+1)\frac{1}{r^2} - \frac{A}{r^2} + \frac{B}{r}\right]R = 0$$

$$\rho = \frac{2\sqrt{-2mE}}{\hbar}r$$

$$\frac{d^2R}{d\rho^2} + \frac{2}{\rho}\frac{dR}{d\rho} + \left(-\frac{1}{4} + \frac{n}{\rho} - \frac{s(s+1)}{\rho^2}\right]R = 0$$

$$\frac{2mA}{\hbar^2} + l(l+1) = s(s+1)$$

$$\frac{B}{\hbar}\sqrt{\frac{m}{-2E}} = n$$

$\rho \ll \Rightarrow \rho^l$

$\rho \gg \Rightarrow e^{-\frac{\rho}{2}} \quad \Rightarrow R = \rho^l e^{-\frac{\rho}{2}} w(\rho)$

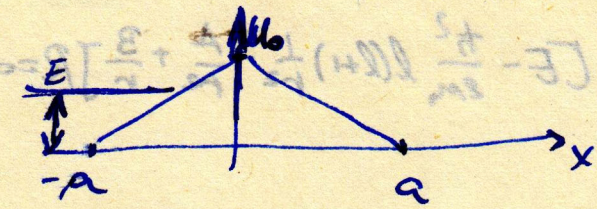
$$\rho w'' + (2s+2-\rho)w' + (n-s-1)w = 0$$

$w = F(-n+s+1, 2s+2, \rho)$ gdje $n-s-1 = p$ cijeli poz.

s. poz. Korijen jbe.

$$-E_p = \frac{2B^2m}{\hbar^2}\left[2p+1+\sqrt{(2l+1)^2 + \frac{8mA}{\hbar^2}}\right]^{-2}$$

$T = ?$ $\quad U(x) = \begin{cases} \alpha x + U_0 & -a < x < 0 \\ -\alpha x + U_0 & 0 < x < a \end{cases}$



$\alpha x + U_0 = 0$

$x = -\frac{U_0}{\alpha}$

$\boxed{\begin{matrix} U_0\left(1+\frac{x}{a}\right) \\ U_0\left(1-\frac{x}{a}\right) \end{matrix}}$

$\left.\begin{matrix} U_0 = 1\,eV \\ \alpha = \frac{4\,eV}{a} \end{matrix}\right\} \frac{U_0}{\alpha} = a$

$$K_0^2 = \frac{2mE}{\hbar^2} \qquad \mathcal{K}_0^2 = \frac{2mU_0}{\hbar^2}$$

$|x| \geq a \qquad \frac{d^2\mu(x)}{dx^2} + K_0^2\,\mu(x) = 0$

$-a \leq x \leq 0 \qquad \frac{d^2u(x)}{dx^2} - \left(\mathcal{K}_0^2 - K_0^2 + \mathcal{K}_0^2\frac{x}{a}\right)u(x) = 0$

$0 \leq x \leq a \qquad \frac{d^2\mu(x)}{dx^2} - \left(\mathcal{K}_0^2 - K_0^2 - \mathcal{K}_0^2\frac{x}{a}\right)\mu(x) = 0$

$$\xi = \left(\frac{a}{\mathcal{K}_0^2}\right)^{4/3}\left(\mathcal{K}_0^2 - K_0^2 + \mathcal{K}_0^2\frac{x}{a}\right)$$

$$\eta = \left(\frac{a}{\mathcal{K}_0^2}\right)^{2/3}\left(\mathcal{K}_0^2 - K_0^2 - \mathcal{K}_0^2\frac{x}{a}\right)$$

$\mu''(x) =$

$$\lambda \equiv k_0^2 a^{4/3} \mathcal{K}_0^{-4/3} = (\mathcal{K}_0 a)^{4/3} \left(\frac{k_0}{\mathcal{K}_0}\right)^2$$

$$\mu \equiv a^{2/3} \mathcal{K}_0^{-4/3} (\mathcal{K}_0^2 - k_0^2) = (\mathcal{K}_0 a)^{4/3} \left(1 - \frac{k_0^2}{\mathcal{K}_0^2}\right)$$

---

ako je brzina čestice iste sa obje strane barijere

tada je koef. prolaza

$$\gamma = |F|^2$$

prop. fje Airy u, v norm.

$$u'(t)v(t) - u(t)v'(t)$$

$$\gamma = |F|^2 = \frac{\lambda}{\{[v(\mu)u'(-\lambda) - u(\mu)v'(-\lambda)]^2 + \lambda [v(\mu)u(-\lambda) - u(\mu)v(-\lambda)]^2}$$

$$\cdot \frac{1}{[v'(\mu)u'(-\lambda) - u'(\mu)v'(-\lambda)]^2 + \lambda[v'(\mu)u(-\lambda) - u'(\mu)v(-\lambda)]^2\}}$$

(1)

$$\frac{d^2u}{d\xi^2} - \xi u = 0$$ Airyeve-ξ u, v

$$\frac{d^2u}{d\eta^2} - \eta u = 0$$

$$u(x) = e^{ik_0x} + A e^{-ik_0x} \qquad x \le -a$$

$$u = B u(\xi) + C v(\xi) \qquad -a \le x \le 0$$

$$u = D u(\eta) + E v(\eta) \qquad 0 \le x \le a$$

$$u = F e^{ik_0x} \qquad x \ge a$$

$x=-a \quad x=0 \quad x=a$

$$e^{-ik_0a} + A e^{ik_0a} = B u(-\lambda) + C v(-\lambda)$$

$$i\sqrt{\lambda}\left(e^{-ik_0a} - A e^{ik_0a}\right) = B u'(-\lambda) + C v'(-\lambda)$$

$$B u(\mu) + C v(\mu) = D u(\mu) + E v(\mu)$$

$$B u'(\mu) + C v'(\mu) = -D u'(\mu) - E v'(\mu)$$

$$D u(-\lambda) + E v(-\lambda) = F e^{ik_0a}$$

$$D u'(-\lambda) + E v'(-\lambda) = -i\sqrt{\lambda} F e^{ik_0a}$$

$|x_{nk}| = |a_{nk}|$ (1)

$[x, p_x] = i\hbar \quad \Rightarrow \quad \sum_{k=0} \frac{2m}{\hbar} \omega_{kn} |x_{nk}|^2 = 1$

$x_{nm} = a_{nm} e^{i\omega_{nm} t}$

$a_{n-k} = a_k(n) \quad \omega_{n-k} = k\omega(n)$

$\omega_{n-k} = k \cdot \omega(n)$

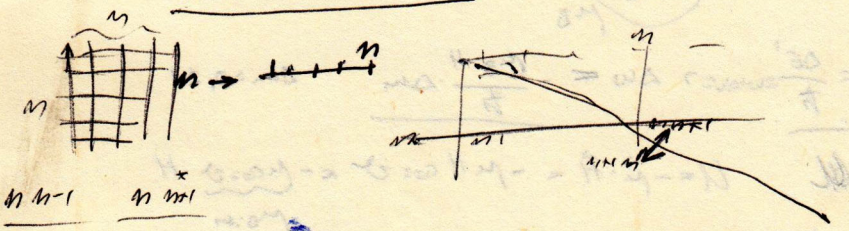
$k = 0, 1, \ldots n-1 \; n$

$n-k = 0, -1, \ldots -(n+1) \; -n$

$n-k = n, n-1, \ldots 0$

$a_{0-1} = 0$

$a_{0-1} = a(0)$



$\sum_{k=-\infty}^{\infty} a_k(n) e^{ik\omega(n)}$

$a_{n-k} \quad \omega_{n-k}$

$k = \infty, 1, \ldots n$

$[q, p] = i\hbar \frac{\partial q}{\partial q} = i\hbar \frac{\partial p}{\partial p}$

$[F, p] = i\hbar \frac{\partial F}{\partial q}$

$[q, F] = i\hbar \frac{\partial F}{\partial p}$

$[\hat{u}, \hat{v}] = i\hbar \{u, v\}$

$\frac{dF}{dt} = \{F, H\} = \left\{ \frac{\partial F}{\partial q} \cdot \frac{\partial H}{\partial p} - \frac{\partial F}{\partial p} \frac{\partial H}{\partial q} \right.$

$= \{F, H\} = \frac{1}{i\hbar} [F, H]$

$[M_x, M_y] = i\hbar M_z$ $\qquad \vec{M} = \hbar \vec{L}$

$[L^2, L_z] = 0$ $\qquad [L_+, L_-] = 2L_z$

$[L^2, L_\pm] = 0$ $\qquad [L_+, L_-] = 2L_z$

$L^2 |lm\rangle = l(l+1) |lm\rangle$

$L_z |lm\rangle = m |lm\rangle$

$(L_z)_{nm} = \langle ln | L_z | lm \rangle = m \delta_{nm}$

$U_{pot} = -\vec{\mu} \cdot \vec{H}$ $\qquad \vec{\mu} = \frac{e}{2m_e c} \vec{M}$ $\qquad M_z = \hbar m$

$$U_{pot} = -\underbrace{\frac{e\hbar}{2m_e c}}_{\mu_B} \cdot m H$$

$$\omega' = \frac{\Delta E'}{\hbar} = \omega + \Delta\omega \Rightarrow \Delta\omega = -\frac{\mu_B \cdot H}{\hbar} \cdot \Delta m \qquad \Delta m = 0, \pm 1$$

$\vec{F} = -\vec{\nabla} U$ $\qquad U = -\vec{\mu} \cdot \vec{H} = -\mu H \cos\vartheta = -\underbrace{\mu \cos\vartheta}_{\mu_B \cdot m} \cdot H$

spin $M_z = S_z = \pm \frac{\hbar}{2}$ ali $\mu_e = \mu_B$

$E = c\sqrt{p^2 + m_0^2 c^2}$ $\qquad \psi(x,t) = A e^{i(kx - \omega t)}$

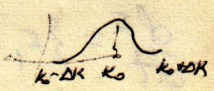
$\left(\frac{E}{c}\right)^2 - p^2 = m_0^2 c^2 \,/ \cdot \hbar^2$ $\qquad \vec{p} = \hbar \cdot \vec{k}$ $\qquad kx - \omega t = \text{const}$

$\left(\frac{E/\hbar}{c}\right)^2 - \frac{p^2}{\hbar} = \frac{m_0^2 c^2}{\hbar^2}$ $\qquad \Rightarrow \frac{dx}{dt} = \frac{\omega}{k} = v_{\text{fazna brzina}}$

$\left(\frac{\omega}{c}\right)^2 - k^2 = \frac{m_0^2 c^2}{\hbar^2}$

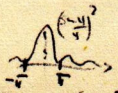
$$\psi(x,t) = \int_{k_0 - \Delta k}^{k_0 + \Delta k} A_1(k) e^{i(kx - \omega t)} dk$$

kvadrat amplitude



$\frac{2\omega d\omega}{c^2} - 2k dk = 0$

$\Rightarrow u = \frac{c^2}{v}$



$y = \Delta k \left(x - t \left(\frac{d\omega}{dk}\right)_{k_0}\right)$

$k = k_0 + \Delta k$ $\qquad A_1(k) \approx A(k_0)$

$\omega(k) = \omega(k_0) + (\Delta k) \left(\frac{d\omega}{dk}\right)_{k_0} + \dots$ $\qquad \Delta k \ll$

$\omega_0 \qquad (k - k_0)$

$\Rightarrow v = \left(\frac{d\omega}{dk}\right)_{k_0}$

$|x_{nk}| = |a_{nk}|$ (1)

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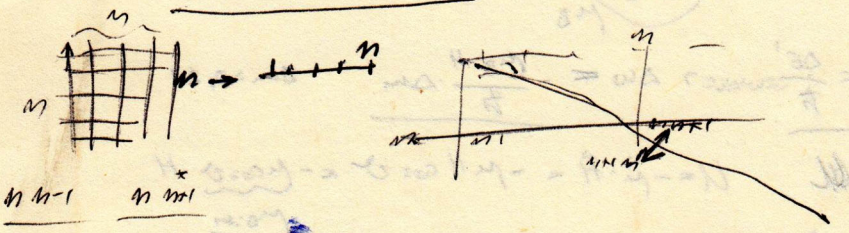
$n-k = n, n-1, \ldots 0$

$a_{0\,-1} = 0$

$a_{0\,-1} = a_1(0)$



$\sum_{k=-\infty}^{\infty} a_k(n) e^{ik\omega(n)}$

$a_{n\,n-k} \quad \omega_{n\,n-k}$

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$= \{F, H\} = \frac{1}{i\hbar} [F, H]$

$\left(\frac{\omega}{c}\right)^2 - k^2 = \left(\frac{m_0 c}{\hbar}\right)^2$ $\quad \psi(x,t) = A e^{i(kx-\omega t)} \Rightarrow -\frac{1}{c^2}\ddot{\psi} + \Delta\psi = \left(\frac{m_0 c}{\hbar}\right)^2 \psi$

$\psi(\vec{r},t) = u(\vec{r}) e^{-i\omega t} \Rightarrow \Delta u(\vec{r}) + \frac{2m}{\hbar^2}(E-U)\, u(\vec{r}) = 0$ $\quad k^2 = \frac{p^2}{\hbar^2} = \frac{2m}{\hbar^2} E$ - k.m.

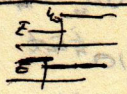
$\vec{p} \to \frac{\hbar}{i}\vec{\nabla}$ $\quad [x, p_x] = i\hbar \Rightarrow \hat{H}\psi = i\hbar\frac{\partial}{\partial t}\psi(\vec{r},t)$ $\quad \psi(\vec{r},t) = u(\vec{r}) e^{-i\frac{E}{\hbar}t}$

diskr. množ. $\quad \psi_n = u_n(\vec{r}) e^{-\frac{i}{\hbar}E_n t} \Rightarrow \psi(\vec{r},t) = \sum_n c_n \psi_n(\vec{r},t)$

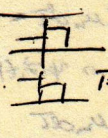
$\vec{\nabla}(\psi\vec{b}) = \vec{\nabla}(\psi)\vec{b} + \vec{\nabla}(\vec{b})\psi = \psi\,\vec{\nabla}\cdot\vec{b} + \vec{b}\cdot\vec{\nabla}\psi$

$\int |\psi|^2 d\tau = 1$ $\quad \frac{\partial\rho}{\partial t} + \operatorname{div}\vec{j} = 0 \Rightarrow \vec{j} = \frac{i\hbar}{2m}(\psi\vec{\nabla}\psi^* - \psi^*\vec{\nabla}\psi)$ $\quad \rho = |\psi|^2$

1) $U=0$ dc kaplor. zlob. zeit. re $E = \frac{\hbar\nu}{\lambda = \frac{h}{p}}$

2) 

$T = 0$?) $\quad T + R = 1$



$T = P = \ominus$

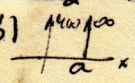
$(a \gg 1)$ $\quad l = \frac{1}{\hbar}\sqrt{2m(U_0 - E)}$ $\quad e^{-\int_{x_1}^{x_2}\frac{2}{\hbar}\sqrt{2m(U(x)-E)}\,dx}$

1-D nema degener. $\quad U(x) = \infty \Rightarrow u(x) = 0$ $\longrightarrow$ i u_1 i u_2 (pro oprom. $\hbar$) $\quad u(a) = 0$ = bez normaln. jako $U(a) \neq \infty$

$u_1'(x)u_2(x) - u_2'(x)u_1(x) = C \Rightarrow u_2(x) = C_1 u_1(x)$

a) $U(x) = U(-x) \Rightarrow u(-x) = \pm u(x)$ parita $\pm$

(b)' $\{\ \}$ - reálný dif. operátor $\Rightarrow u(x) = e^{i\varphi} u^*(x)$
& kvantiz. energie

b)  $\Rightarrow E_n = \frac{\hbar^2\pi^2}{2ma^2}\cdot n^2$ $\quad u_n(x) = A\sin\frac{n\pi}{a}x$ $\quad A = \sqrt{\frac{2}{a}}$

c) $\quad u(x) = \pm u(-x)$ $\Rightarrow |U_0| - |E| = \frac{2\hbar^2}{ma^2}(\varphi_k)^2$ $\quad \frac{\sin}{\cos}\ \operatorname{tg}\varphi = \frac{\sqrt{k^2-\varphi^2}}{\varphi}$ $\quad \alpha^2 = \frac{m|U_0|}{2\hbar^2}a^2$

$\operatorname{cotg}\varphi = -\frac{\varphi}{\sqrt{k^2-\varphi^2}}$ $\quad \varphi = \frac{ka}{2}$

H.O. $\quad U(x) = \frac{m\omega^2}{2}x^2$ $\quad a = \sqrt{\frac{\hbar}{m\omega}}$ $\quad E = \frac{\hbar\omega}{2}\cdot K$ $\quad s = \frac{x}{a} \Rightarrow u_1(s) = e^{-\frac{s^2}{2}} H_n(s)$ $\quad \phi(s,t) = e^{2st - t^2} = \sum_{n=0}^{\infty} H_n(s)\frac{t^n}{n!}$

$H_n'' - 2sH_n' + \frac{(K-1)}{2n}H_n = 0 \Rightarrow K = 2n+1$

III Trim. stredný $\quad U(x,y,z) = U_x(x) + U_y(y) + U_z(z) \Rightarrow \frac{d^2u_x}{dx^2} + \frac{2m}{\hbar^2}(E_x - U_x)u_x = 0$

$U = U(r)$ $\quad \Delta = \frac{1}{r}\left(\frac{\partial^2}{\partial r^2}(r\ \ )\right) + \frac{1}{r^2}\Lambda$ $\quad \Lambda = \frac{1}{\sin\vartheta}\frac{\partial}{\partial\vartheta}\left(\sin\vartheta\frac{\partial}{\partial\vartheta}\right) + \frac{1}{\sin^2\vartheta}\frac{\partial^2}{\partial\varphi^2}$

$u(r,\vartheta,\varphi) = R(r)\,Y(\vartheta,\varphi)$

$M^2 = -\hbar^2\Lambda$ $\quad \Rightarrow \Lambda Y = \lambda Y$ $\quad \frac{1}{r}\frac{d^2}{dr^2}(rR) + \left[\frac{2m}{\hbar^2}(E-U) + \frac{\lambda}{r^2}\right]R = 0$ $\quad \lambda = -l(l+1)$

$\Rightarrow Y_l^m(\vartheta,\varphi) = \sqrt{\frac{2l+1}{4\pi}\frac{(l-|m|)!}{(l+|m|)!}}\,P_l^m(\cos\vartheta)\,e^{im\varphi}$ $\quad P_l^m(\cos\vartheta) = \sin^m\vartheta\,\frac{d^m}{d\cos\vartheta}P_l(\cos\vartheta)$

$|m| \leq l$

$\int_0^{2\pi}\int_0^{\pi} Y_l^{m*} Y_{l'}^{m'}\,\underbrace{d\varphi\sin\vartheta\,d\vartheta}_{d\Omega} = \delta_{mm'}\delta_{ll'}$ $\quad P_l^m(t) = (1-t^2)^{\frac{|m|}{2}}\left(\frac{d}{dt}\right)^{|m|}P_l(t)$

Parita $\quad \vec{r} \to -\vec{r}$ $\quad \begin{cases} r \to r \\ \varphi \to \varphi + \pi \\ \vartheta \to \pi - \vartheta \end{cases}$ $\quad Y_l^m \to (-1)^l Y_l^m$ $\quad P_l(t) = \frac{1}{2^l l!}\left(\frac{d}{dt}\right)^l (t^2-1)^l$

$\rho = r\frac{2}{\hbar}\sqrt{2m|E|} \Rightarrow \left[\frac{d^2}{d\rho^2} - \frac{1}{4} + \frac{n}{\rho} - \frac{l(l+1)}{\rho^2}\right]v(\rho) = 0$ $\quad v(r) = rR(r)$ $\quad |E| = \frac{me^2e^4}{2\hbar^2 n^2}$ $\quad NaF = \frac{\hbar^2}{me^2}$

$\rho \to \infty$ $\quad \rho \to 0$ $\quad \Rightarrow v(\rho) = e^{-\rho}\rho^{l+1}L(\rho)$

$\Rightarrow \rho L''(\rho) + [2(l+1)-\rho]L'(\rho) + (n-l-1)L(\rho) = 0 \qquad L(\rho) = \sum_{k=0}^{n_r} a_k \rho^k \quad \Rightarrow n_r$ - konečen, inače $L(\rho) \to e^{\rho}$

$\Rightarrow n_r + l + 1 - n = 0 \quad \Rightarrow n = 1, 2, 3, \dots$

$$u_{nlm}(r,\vartheta,\varphi) = R_{nl}(r)\, Y_l^m(\vartheta,\varphi) \qquad R_{nl}(r) = -\sqrt{\left(\frac{2Z}{r_B n}\right)^3 \frac{n_r!}{2n[(n+l)!]^3}}\, e^{-\frac{\rho}{2}} \rho^l L_{n+l}^{2l+1}(\rho)$$

vodík: $Z=1$, $l=0$, $m=0$, $n_r=0$

$$u_{100}(r,\vartheta,\varphi) = \frac{1}{\sqrt{\pi r_B^3}} e^{-\frac{r}{r_B}} \qquad \rho = |u|^2 \Rightarrow d\omega = \rho\, d\tau \Rightarrow d\omega_r = \frac{4}{r_B^3} e^{-\frac{2r}{r_B}} r^2 dr \Rightarrow r_w = r_B$$

$\frac{(Z\alpha)^2}{n^2} \Rightarrow Z = 1 + \sqrt{\alpha\omega}$

$$(E_n - E_m)\int u_m^* u_n\, d\tau = -\frac{\hbar^2}{2m}\int_S (u_n^* \vec{\nabla} u_n - u_n \vec{\nabla} u_m^*)\, d\vec{f} = 0 \Rightarrow \int u_n^* u_m\, d\tau = \delta_{nm}$$

$\to$ Hamilt. stacionární $\Rightarrow \psi(\vec{r},t) = \sum_n c_n u_n(\vec{r}) e^{-\frac{i}{\hbar}E_n t} \qquad c_n = \int u_n^* \psi(\vec{r},0)\, d\tau \qquad \int \psi^2 d\tau = 1 \Rightarrow \sum_n |c_n|^2 = 1$

$$G_{nm} = \int u_n^* \hat{G} u_m\, d\tau \qquad (GF)_{nm} = \int u_n^* \hat{G}\hat{F} u_m\, d\tau \qquad \hat{F} u_m = \sum_l c_l u_l \Rightarrow c_l = \int u_l^* \hat{F} u_m = F_{lm}$$

a $(\hat{p}^2 - 2\hat{p})_{nm} = -i\hbar\delta_{nm} \qquad H_{nm} = E_n \delta_{nm}$

Postulat $\bar{G} = \int \psi^* \hat{G} \psi\, d\tau \qquad \bar{G} = \sum_n \sum_m c_n^* c_m e^{i\omega_{nm} t} G_{nm} \qquad \bar{G} = \bar{G}^* \Rightarrow \boxed{G_{nm} = G_{mn}^*}$ Hermit. matrice

reálná fyz. veličina

$\boxed{\hat{G} u_n = g_n u_n} \Rightarrow G_{nm} = g_n \delta_{nm} \Rightarrow \bar{G} = \sum_n |c_n|^2 g_n$ tj. $\bar{G} = \int u^*(\vec{r}) \hat{G} u(\vec{r})\, d\tau \qquad u(\vec{r}) = \psi(\vec{r},0)$

$$(\Delta G)^2 = \overline{(G-\bar{G})^2} = \overline{G^2} - \bar{G}^2 = 0$$

$\hat{M}_z = \frac{\hbar}{i}\frac{\partial}{\partial\varphi} \qquad M^2 = -\hbar^2\Delta$

Variační postup: $E_n = \int u_n^* \hat{H} u_n\, d\tau \qquad \int |u_n|^2 d\tau = 1 \Rightarrow \mathcal{E} = \int \phi^* \hat{H} \phi\, d\tau \qquad \int \phi^* \phi\, d\tau = 1 \Rightarrow \mathcal{E} \geq E_0$

$\phi(\vec{r}) \qquad \phi = \sum_n a_n u_n \Rightarrow \sum_n |a_n|^2 = 1 \Rightarrow \mathcal{E} = \sum_n |a_n|^2 E_n \qquad \mathcal{E} - E_0 = \sum_n |a_n|^2 (E_n - E_0) \geq 0$

$\hat{H}\psi = i\hbar\frac{\partial\psi}{\partial t} \qquad \hat{H} = -\frac{\hbar^2}{2}\sum_i \frac{\Delta_i}{m_i} + U \qquad \hat{H}(1,2) = \hat{H}(2,1) \qquad |\psi(1,2)|^2 = |\psi(2,1)|^2 \Rightarrow \psi(2,1) = \pm\psi(1,2)$ kod permutacyje 2 částic

nemá interakce, $\hat{H}(1,2) = \hat{H}(1) + \hat{H}(2) \Rightarrow$ řešení $\psi_a(1)\psi_b(2)$ & $\psi_b(1)\psi_a(2) \Rightarrow \psi(1,2) = \frac{1}{\sqrt{2}}[\psi_a(1)\psi_b(2) \pm \psi_b(1)\psi_a(2)]$ Bos. sym. / Ferm. asym.

$\Rightarrow \psi_{as}(1,2) = \frac{1}{\sqrt{2}}\begin{vmatrix} \psi_a(1) & \psi_a(2) \\ \psi_b(1) & \psi_b(2) \end{vmatrix} = 0$ za $\psi_a(1) = \psi_b(1)$ tj. a, b část. ve jed. st.

Fermioni

H.O. $\bar{E} = \frac{\hbar\omega}{2} + \frac{\hbar\omega}{e^{\frac{\hbar\omega}{kT}} - 1} \qquad$ iz $E_N = \frac{\hbar\omega}{2}(2N+1) \quad \frac{dN}{d\omega} = \mu_\omega = \frac{\omega^2 \bar{E}}{\pi^2 c^3} = \frac{\hbar\omega^3}{\pi^2 c^3} \cdot \frac{1}{e^{\frac{\hbar\omega}{kT}} - 1}$ Planck-ův zákon $\quad \mu_\lambda = \left|\frac{d\mu}{d\lambda}\right|$

$Z = \sum_{N=0}^{\infty} e^{-\beta E_N} \qquad U = V\int_0^\infty \mu_\omega\, d\omega = aVT^4 \qquad F = 2\sum_{\vec{l}} f(\omega) = kT\frac{V}{\pi^2 c^3}\int_0^\infty d\omega\, \omega^2 \ln(1 - e^{-\beta\hbar\omega}) \qquad e^{-\frac{\hbar\omega}{kT}} < 1 \qquad \ln(1-x) = \Sigma$

$Z = \frac{1}{1 - e^{-\beta\hbar\omega}} \qquad \bar{E} = \hbar\omega(\bar{N}_\omega) \qquad f(\omega) = -kT\ln Z \qquad N_f = 2\sum_{\vec{l}} N(\omega) = -\left(\frac{\partial\beta F}{\partial\nu}\right)_{\beta,V} \qquad U = 2\sum_{\vec{l}} E(\omega) = \left(\frac{\partial\beta F}{\partial\beta}\right)_{\nu,V}$

Krystal. řešetka $3N$ stupňů $\Rightarrow 3N$ slob. $\Rightarrow F = 3\sum_{\vec{l}} f(\omega)$ sadrží fyzikál. vlastnosti $= 3kT\sum_{\vec{l}} \ln(1 - e^{-\beta\hbar\omega})$

problém $\Sigma \to \int \quad \omega = f(\vec{l})$

a) Einstein. - suroučil. ist. $\omega \Rightarrow F = 3NkT\ln(1 - e^{-\beta\hbar\omega}) \qquad U = 3N\bar{E}(\omega) \qquad N_f = 3N\bar{N}(\omega)$

b) Debye $\omega = \mu\vec{l}$ za $l \ll$ tj. $\lambda \gg$ aprox. nahrazení za malé ω tj. $\hbar\omega \ll kT$ pro H.O. nejdůležitější, doprinos napravit za $kT \gtrsim \hbar\omega \qquad \Rightarrow \Sigma \to \int_0^{\omega_m} \frac{V}{\pi^2 c^3}\omega^2\, d\omega$ ω_m - gorniza frekvencí

$$F = 3KT \sum_{z} \ln(1-e^{-\beta\hbar\omega})_{\nu=0} = 3KT \frac{V}{2\pi^2 \mu^3} \int_0^{\omega_m} \omega^2 \ln(1-e^{-\beta\hbar\omega})_{\nu=0} d\omega$$

ω_m – pri kadnrok. h T pobudim. zamo N-dno spektra

$\boxed{K\Theta = \hbar\omega_m}$ $\Rightarrow F = 9NKT \left(\frac{T}{\Theta}\right)^3 \int_0^{\Theta/T} x^2 \ln(1-e^{-x})_{\nu=0} dx$

a) $T \gg \Theta$ $U = 3NKT$ b) $\Theta \gg T$ korektno, ker aprox. najbolj / $\sim KT \ll \hbar\omega \lesssim K\Theta$ $\Rightarrow \int_0^\infty$

$1-e^{-x} = x - \nu$ $F = 18N\zeta(4)\, KT \left(\frac{T}{\Theta}\right)^3$ $U = \frac{3\pi^4 NKT}{5} \left(\frac{T}{\Theta}\right)^3$

2A-molek. $E = \frac{\hbar^2 \pi^2}{2(m_1+m_2)d^2}(n_1^2+n_2^2+n_3^2)$ $T = 10^{-14}$ $\boxed{\text{Vrtenje 1.}}$

$\boxed{\text{rot.}}$ $E = \frac{\mu^2}{2J} = \frac{\hbar^2 l(l+1)}{2J}$ $F = 150$ $J = \mu r^2$ $\boxed{\frac{\hbar^2}{2J} = K \cdot \Theta_r}$

$x_l = \frac{\Theta_r}{T}(l+1)l$

$\Delta x_l = x_{l+1} - x_l = x_l$

$Z = \sum_{l=0}^{\infty} (2l+1) e^{-\frac{\Theta_r}{K} l(l+1)}$ $F = -NKT \ln Z$ a) $T \gg \Theta_r$ $\Rightarrow U = NKT$ $\Sigma \to \frac{T}{\Theta_r}\int_0^\infty e^{-x} dx$

b) $T \ll \Theta_r$ $Z \approx 1 + 3e^{-2\frac{\Theta_r}{T}}$

$\Rightarrow F = -3NKT e^{-2\frac{\Theta_r}{T}}$ $U = 6NK\Theta_r e^{-2\frac{\Theta_r}{T}}$

$\boxed{\text{osc.}}$ $\Theta_{osc} = \frac{\hbar\omega}{K}$

$\delta(\ln B - \alpha N - \beta U) = 0$ $S = K \ln B$ $\Rightarrow K\alpha = \left(\frac{\partial S}{\partial N}\right)_{U,N}$ $K\beta = \left(\frac{\partial S}{\partial U}\right)_{V,N}$

$T\delta s = \delta u + P\delta v$ $\Rightarrow T\delta S = \delta U + P\delta V - (\underbrace{\mu + Pv - Ts}_{\mu})\delta N$ $G = N \cdot \mu$

$\alpha = -\frac{\mu}{KT}$, $\beta = \frac{1}{KT}$

Fotoni: ravnotežje $T_1 = T_2$ $P_1 = P_2$ $\mu_2 = \mu_1$

$S = K(\Omega + \beta U - \nu N)$ $\Omega = \pm \sum_i g_i \ln[1 \pm e^{-\beta E_i}]$ (F) (B) velikokan. potencial iz izraza za $\ln B$

$P = KT\left(\frac{\partial \Omega}{\partial V}\right)_{\beta,\nu}$ $U = -\left(\frac{\partial \Omega}{\partial \beta}\right)_{V,\nu}$ $N = \left(\frac{\partial \Omega}{\partial \nu}\right)_{V,\beta}$

Fotoni, fononi: $\alpha = 0 \Rightarrow \nu = 0$ $E_i = \hbar\omega$

$\Omega = -\sum_{z,\sigma} \ln(1-e^{-\beta\hbar\omega})_{\nu=0}$ $S = K(\Omega + \beta U - \underset{0}{\nu} N) \Rightarrow F = -KT\Omega = KT \sum_{z,\sigma} \ln(1-e^{-\beta\hbar\omega})_{\nu=0}$

Vezani sklas $e^{\nu} \ll 1$ zaprav $e^{\alpha+\beta E_i} \gg 1$ $\Rightarrow \Omega = e^{\nu} \underbrace{\sum_i g_i e^{-\beta E_i}}_{Z}$ iz $\ln(1 \pm x) =$

$N = \left(\frac{\partial \Omega}{\partial \nu}\right)_{V,\beta} = e^{\nu} \cdot Z$ $\Rightarrow U = -\left(\frac{\partial \Omega}{\partial \beta}\right)_{V,\nu} = -N\left(\frac{\partial \ln Z}{\partial \beta}\right)_{V,\nu}$

$F = -NKT \ln Z$ $\Rightarrow U = \left(\frac{\partial \beta F}{\partial \beta}\right)_{V,\nu}$

$\Rightarrow U = F - T\left(\frac{\partial F}{\partial T}\right)_V$

$F + TS$

1-A idealni plin $\sum_i g_i \to \frac{2s+1}{h^3}\int d\phi = \int_0^\infty g(E)\, dE$ $g(E) = \underbrace{\frac{2s+1}{h^3} 2\pi V (2m)^{3/2}}_{\gamma} \sqrt{E}$

$\Rightarrow \Omega = \pm g \cdot \beta^{-3/2} \int_0^\infty \sqrt{t} \ln(1 \pm e^{-t}) dt$

$\Rightarrow U = \frac{3}{2} PV$ kot klas.

za $\frac{1}{1-z} = 1+z+z^2+\dots$

Klas. limes

$e^{\alpha+\beta E_i} \gg 1 \Rightarrow g_i = e^{\frac{\mu-E_i}{kT}}\left[1 \mp e^{\frac{\mu-E_i}{kT}} + \dots\right]$ (bozon.)

$\alpha \ll 1$ $\quad N_i \ll g_i$

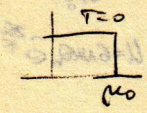
$\lambda = \frac{h}{p}$

$N = \sum_i g_i \cdot g_i \to g \cdot (kT)^{3/2} \int_0^\infty \frac{\sqrt{t}\,dt}{e^{t-\alpha}\pm 1} = e^{\alpha} \cdot \int_0^\infty t^{1/2} e^{-t} dt \Rightarrow \alpha = e^{\alpha} = \frac{Nh^3}{(2S+1)\,V(2\pi m kT)^{3/2}}$

$\alpha \gg 1$ degenerat. plyn

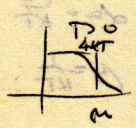
Degen. Fermion.

$\mu(T) \gg kT \Rightarrow T \ll T_F = \frac{\mu_0}{k}$ $\quad a \approx \left(\frac{T_F}{T}\right)^{3/2} \gg 1$



$N = \int_0^\infty g(E)\,g(E)\,dE = g\int_0^\infty g(E)\sqrt{E}\,dE \Rightarrow \mu_0 = \left[\frac{3\pi^2 N}{4\pi V(2S+1)}\right]^{2/3} \cdot \frac{1}{2m}$

$U_0 = \int_0^\infty g(E)\,g(E)\,E\,dE \Rightarrow U_0 = \frac{3}{5}N\mu_0$ $\quad$ iz $U_0 = \frac{3}{2}P_0 V$ $\Rightarrow P_0 \xrightarrow{h\to 0} 0$



$\Delta E \approx kT \ll \mu(T)$ jaka prompera g

$\Delta U = N_{eff} \cdot kT$ $\quad \frac{N_{eff}}{N} = \frac{kT}{\mu_0} \Rightarrow \Delta U = N\frac{(kT)^2}{\mu_0} \Rightarrow C_V \simeq Nk\frac{kT}{\mu_0} \to 0 \quad T\to 0$

Sommerfeld

$N = \int_0^\infty g(E)\,g(E)\,dE$

$U = \int_0^\infty g(E)\,E\,g(E)\,dE$

$\int_0^\infty g(E)\frac{\partial\varphi}{\partial E}dE \overset{\varphi(0)=0}{=} -\int_0^\infty \varphi(E)\frac{dg}{dE}dE = \varphi(\mu) + 2\sum_{n=1}^\infty (1-2^{1-2n})\zeta(2n)(kT)^{2n}\varphi^{(2n)}(\mu)$

$= \varphi(\mu) + \frac{\pi^2}{6}(kT)^2\varphi^{(2)}(\mu) + \frac{7\pi^4}{360}(kT)^4\varphi^{(4)}(\mu) + \dots$

$\varphi(E) = \sum_{n=0}^\infty \frac{\varphi^{(n)}(\mu)}{n!}(E-\mu)^n$

$\frac{dg}{dE}$ - szer. prawa μ $\Rightarrow \sum_{n=0} (E-\mu)^{2n}$ $\quad \to I = \varphi(\mu) - 2\sum_{l=1}^\infty \frac{\varphi^{(2l)}(\mu)}{(2l)!}\int_\mu^\infty (E-\mu)^{2l}\frac{dg}{dE}dE$

$N = \frac{2}{3}\gamma\mu_0^{3/2}$

iz N dobieramy rozw. $\mu = \mu_0\left(1 - \frac{\pi^2}{12}\left(\frac{kT}{\mu_0}\right)^2 \dots\right)$

a zatem binomi. rozwoj kod U

$E-\mu = (kT)x$ subst. $\Rightarrow \int_0^\infty x^{2l}\frac{dg}{dx}dx$

$\Rightarrow g(E) = \frac{e^{-x}}{1+e^{-x}}$ rozw. rod

parcj. indoprnaj

$$U = \frac{3}{5}N\mu_0\left[1 + \frac{5\pi^2}{12}\left(\frac{kT}{\mu_0}\right)^2 \dots\right]$$

Degen. Bozon.

$t = \beta E$

$N = \underbrace{\gamma}_{(1)}\underbrace{(kT)^{3/2}}_{(2)}\int_0^\infty \frac{\sqrt{t}}{e^{t-\alpha}-1}dt$ $\quad$ za Bozone $e^{\alpha} \geq 1 \Rightarrow e^{\alpha} < 1$, $e^{\alpha+\beta E_i} \Rightarrow e^{\alpha-t} < 1$

$= \frac{\sqrt{\pi}}{2}\sum_{n=1}^\infty \frac{e^{n\alpha}}{n^{3/2}}$ $\quad e^{n\alpha} = e^{\frac{n|\mu|}{kT}}$ $\quad \mu < 0$

$T\searrow \Rightarrow (1)\searrow (2)\nearrow \Rightarrow |\mu(T)|\nearrow$ $\quad \uparrow^{0}\ \mu(T)$ $\quad \mu_{max}$ za $T = T_c$

za $T < T_c$ za $E=0$ pogrestka kod $\Sigma \to \int$

$\Rightarrow N = N_0 + N$

za $E=0$: $N_0 = \frac{2S+1}{e^{\frac{\mu}{kT}}-1}$ $\quad N_p = N\left(\frac{T}{T_c}\right)^{3/2}$ za $T<T_c$ $\quad T_c = \frac{1}{2\pi m k}\left[\frac{Nh^3}{(2S+1)V\,2{,}612}\right]^{2/3}$

$U = N_p \cdot kT \sim T^{5/2}$ $\quad a = e^{\alpha} = \left(\frac{T_c}{T}\right)^{3/2} \gg 1$